Please Note

These pdf slides are configured for viewing on a computer screen.

Viewing them on hand-held devices may be difficult as they require a "slideshow" mode.

Do not try to print them out as there are many more pages than the number of slides listed at the bottom right of each screen.

Apologies for any inconvenience.

Domain and Range of Functions

Numeracy Workshop

geoff.coates@uwa.edu.au

STUDYSmarter
Learning, Language and Research Skills

Introduction

This workshop explores **functions** further. The emphasis will be on finding the **domain** and **range** of a given function and we will introduce **function composition**.

Drop-in Study Sessions: Monday, Wednesday, Thursday, 10am-12pm, Meeting Room 2204, Second Floor, Social Sciences South Building, **every week.**

Website: Slides, notes, worksheets.

http://www.studysmarter.uwa.edu.au → Numeracy → Online Resources

Email: *geoff.coates@uwa.edu.au*

Workshops coming up

Week 7: Tuesday 16/4 (12-12.45pm): Functions and transformations

Week 8: Friday 26/4 (1-1.45pm): Fixing your maths mistakes

Domain of a Function

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Alternatively, we can write the domain in interval notation:

$$D = [0, \infty).$$

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Look out for division and square roots!

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The range can be **significantly** harder to work out than the domain.

Domain and Range: Examples

Consider the function $f(x) = x^2 + 2$

What is the domain of f?

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Option: We might recognise that the "x^2" term always returns a number ≥ 0 so the smallest output must be $0^2 + 2 = 2$.

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Consider the function $f(x) = x^2 + 2$

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What is the range?

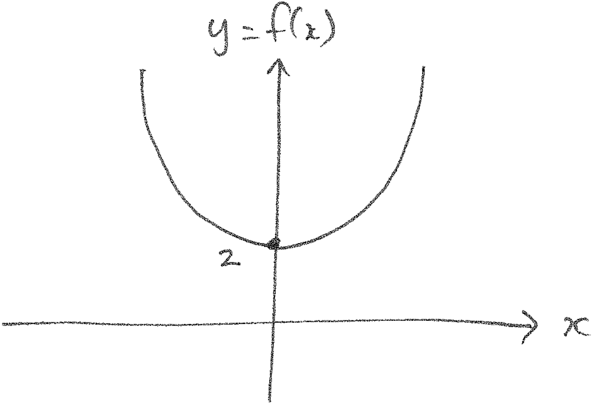
Option: We might recognise that the "x^2" term always returns a number ≥ 0 so the smallest output must be $0^2 + 2 = 2$.

Option: We may recognise the function as a quadratic which produces a parabolic graph (whose turning point/minimum occurs when $y = f(x) = 2$).

$$R = \{x \in \mathbb{R} : x \geq 2\} = [2, \infty)$$

Domain and Range: Examples

Tip: This notion of domain and range can be made clearer by examining the graph of $y = x^2 + 2$.



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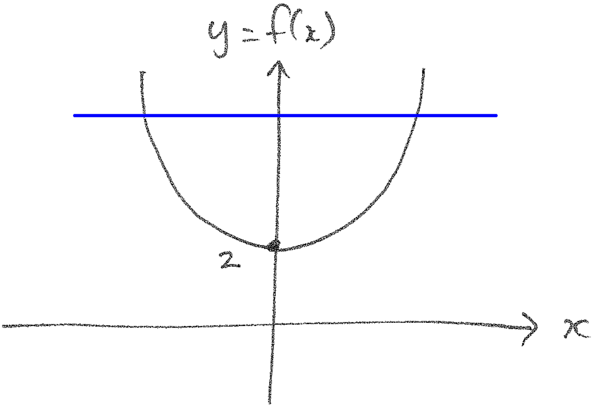

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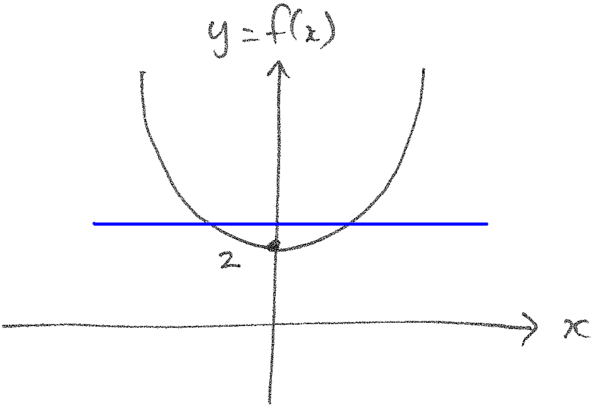

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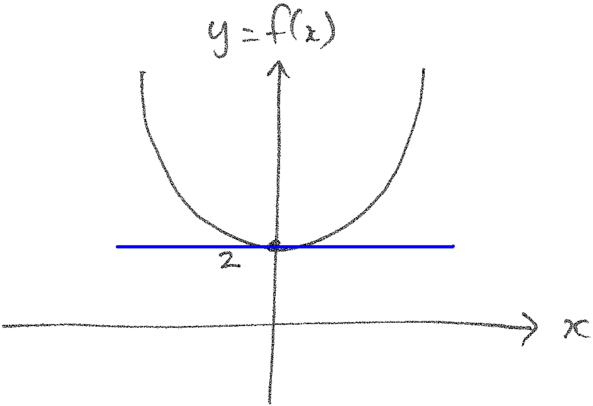

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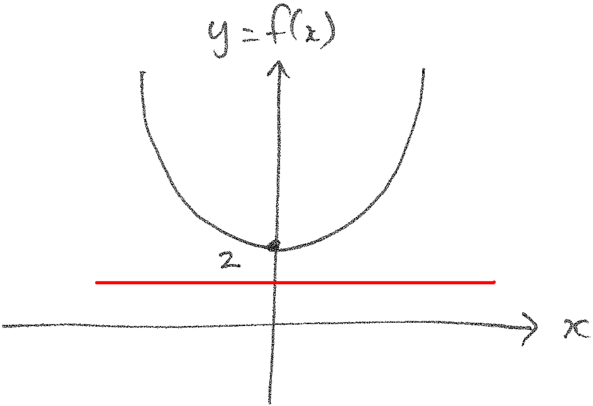

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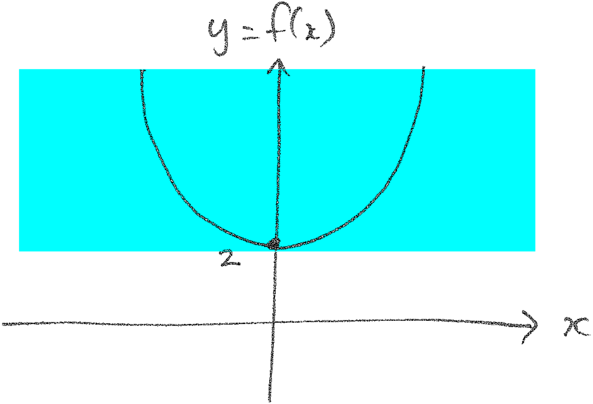

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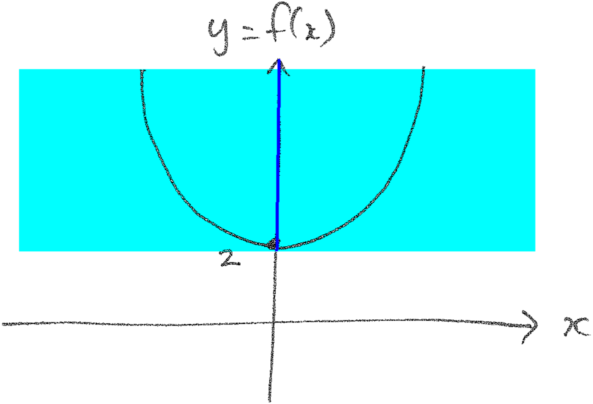

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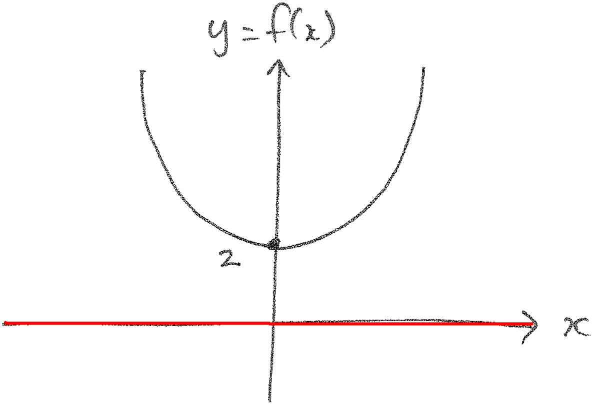

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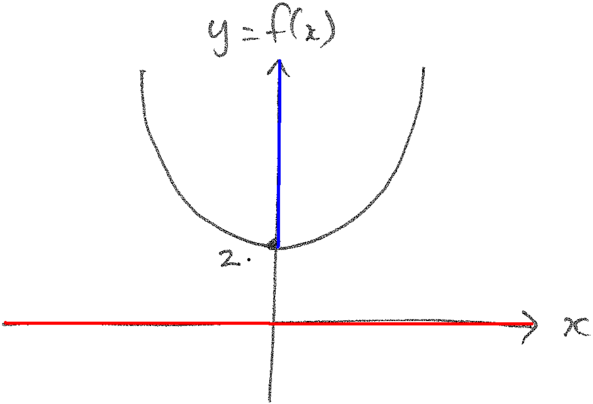

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The range is simply all of the points which lie left/right of the curve.

Function Composition

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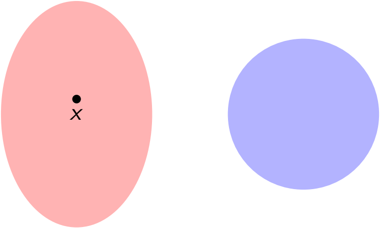

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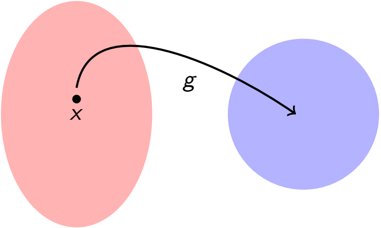

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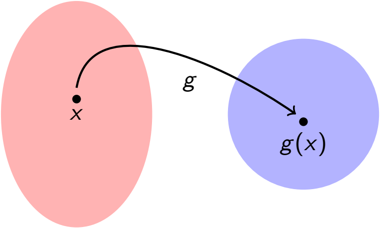

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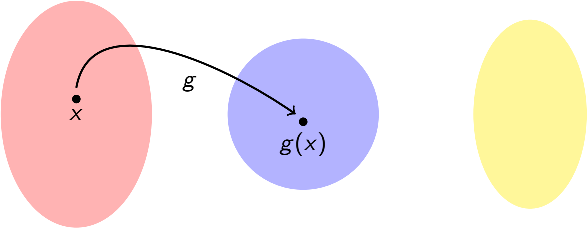

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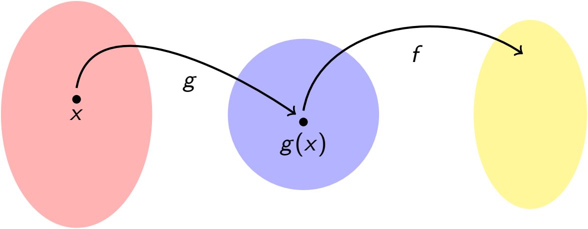

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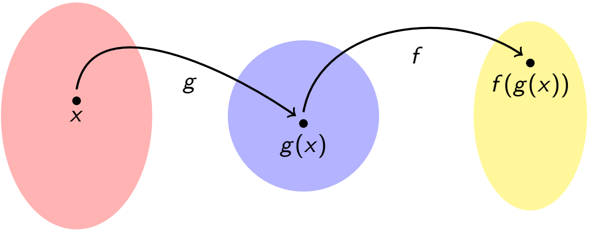

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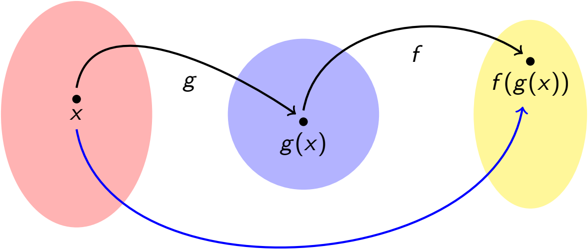

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$$f(x) = x^2$$

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There are two ways in which we can compose these functions, by doing one first and then the other.

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It's tempting to simplify this function:

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but this makes it harder to answer the next question.

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but this makes it harder to answer the next question.

(ii) What is the domain of $(g \bigcirc f)$?

The structure $(g \bigcirc f)(x)$ includes $\sqrt{x-4}$, even though it doesn't appear in the simplified version. This means that the domain of $(g \bigcirc f)(x)$ is also

$$[4, \infty)$$

Function Composition

(iii) Find the range of $(g \bigcirc f)$.

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The simplified version of this function makes the range easy to find:

$$(g \bigcirc f)(x) = x$$

but remember that only $x \geq 4$ are allowable inputs.

Function Composition

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The simplified version of this function makes the range easy to find:

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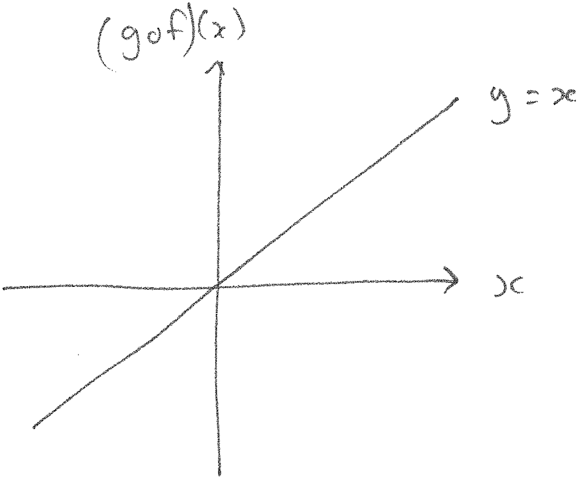
but remember that only $x \geq 4$ are allowable inputs.

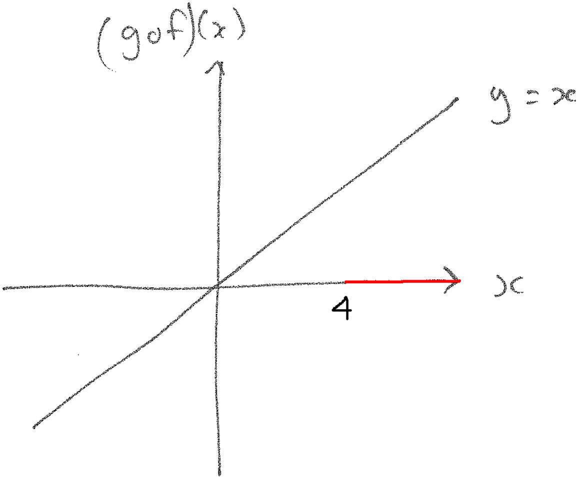
Hence, the range is

$$[4, \infty)$$

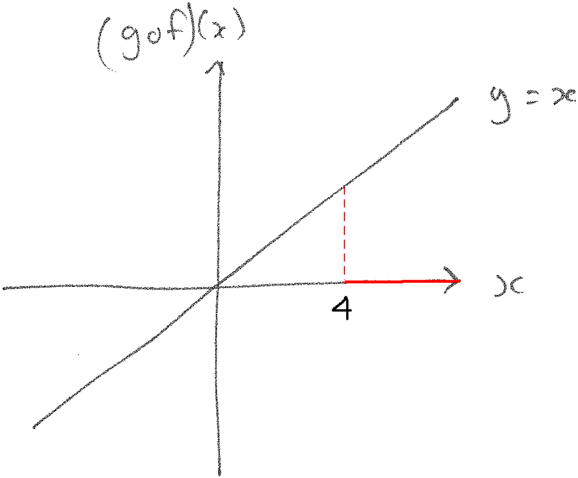
Function Composition



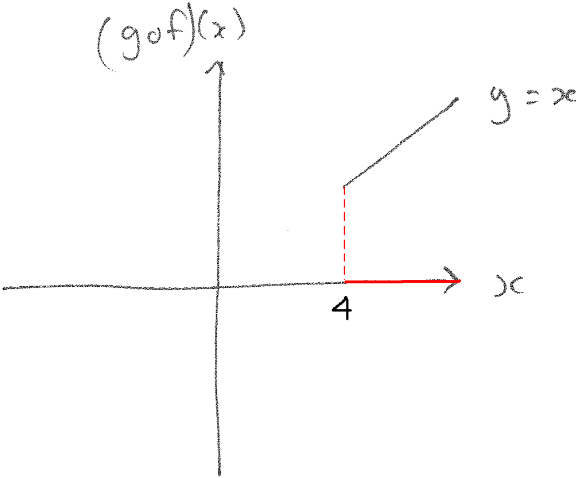
Function Composition



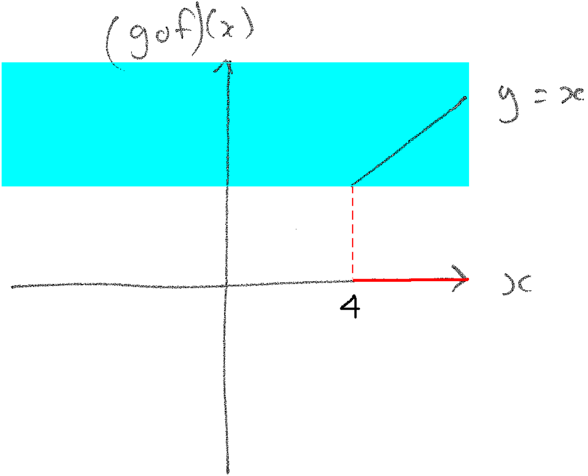
Function Composition



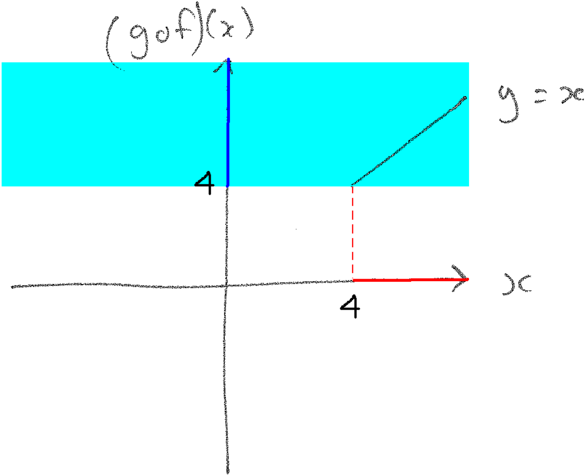
Function Composition



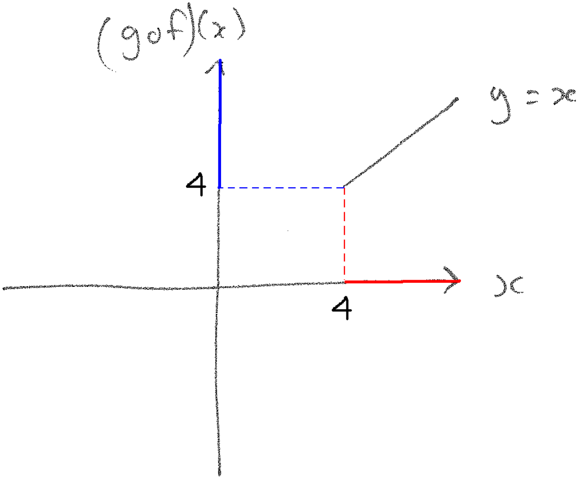
Function Composition



Function Composition



Function Composition



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