

## **Please Note**

These pdf slides are configured for viewing on a computer screen.

**Viewing them on hand-held devices may be difficult as they require a “slideshow” mode.**

**Do not try to print them out as there are many more pages than the number of slides listed at the bottom right of each screen.**

Apologies for any inconvenience.

# Introduction to Functions

## Numeracy Workshop

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# Introduction

Here we introduce and gain confidence with **functions** and their graphs.

**Drop-in Study Sessions:** Monday, Wednesday, Thursday, 10am-12pm, Meeting Room 2204, Second Floor, Social Sciences South Building, **every week**.

**Website:** Slides, notes, worksheets.

<http://www.studysmarter.uwa.edu.au> → Numeracy → Online Resources

**Email:** *geoff.coates@uwa.edu.au*

Workshops that follow this one

Week 6: Tuesday 9/4 (12-12.45pm): Domain and range

Week 7: Tuesday 16/4 (12-12.45pm): Functions and transformations

# Functions

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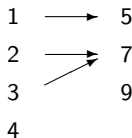


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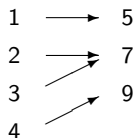


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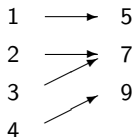


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This is just one example of a function between  $A$  and  $B$ .

## What makes a rule a function?

- It sends each number in a set (maps it to a new number) in a (possibly different) set.

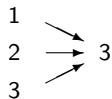
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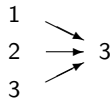
- It sends each number in a set (maps it to a new number) in a (possibly different) set.
- **Key Point:** Functions do **not** send a single number to *two different numbers*.  
(However, you can send two different numbers to the same number.)

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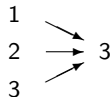


Yes, because no single element gets sent to two different elements.



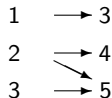
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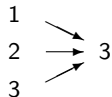
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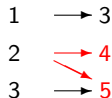
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No, because 2 gets sent to both 4 and 5.

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Then, instead of writing  $1 \rightarrow 3$ , we can write  $f(1) = 3$ .

The notation is standard:

$$f(\text{input}) = \text{output}$$

**Note:** We are used to " $f(x)$ " standing for " $f \times x$ " where  $f$  and  $x$  are variables. If we really wanted to say this, we would write " $fx$ ".

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We use equations instead.



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Here, **x** is the **input**, and  **$2x^2$**  is the **output**.

This allows us to specify the **image** (**output**) of any real number (**input**).

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Clearly, this rule does not send any number to two outputs.

# Graphing functions

It is often useful to represent function rules graphically on the **Cartesian Plane** (or  $(x, y)$  co-ordinate system).

If you are already familiar with drawing the graphs of functions such as  $f(x) = x^2$ , click [▶ here](#) to skip this section.

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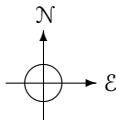
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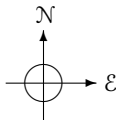
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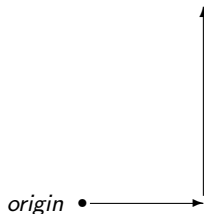
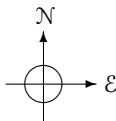
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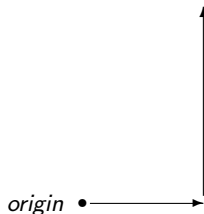
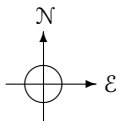
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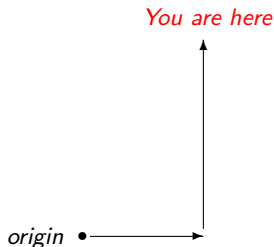
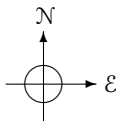
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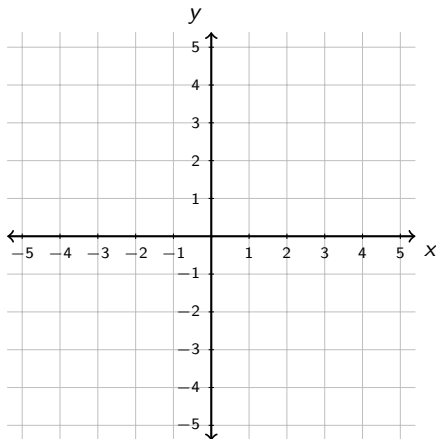
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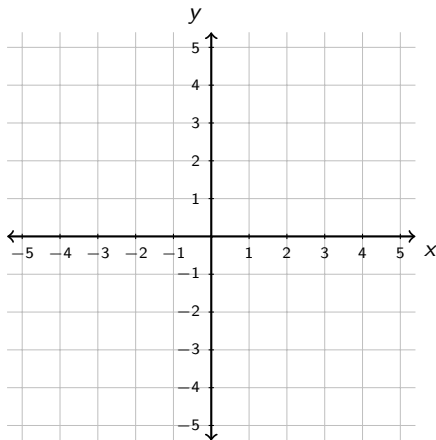
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The most commonly used **two-dimensional coordinate system** is the **Cartesian plane**.



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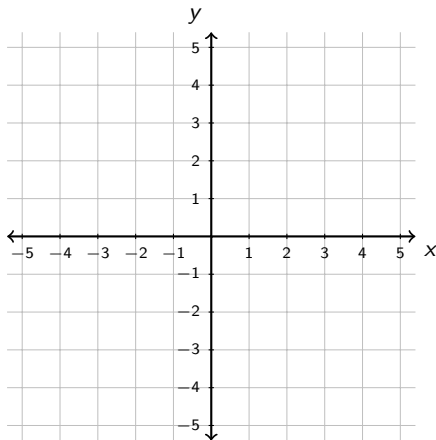
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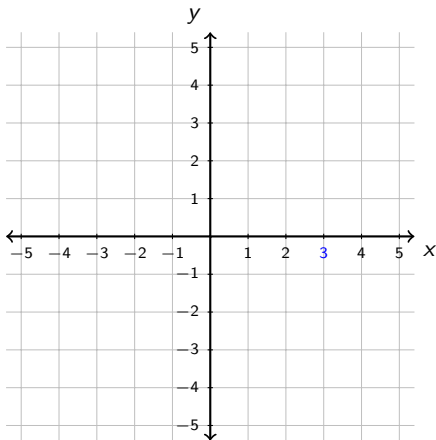
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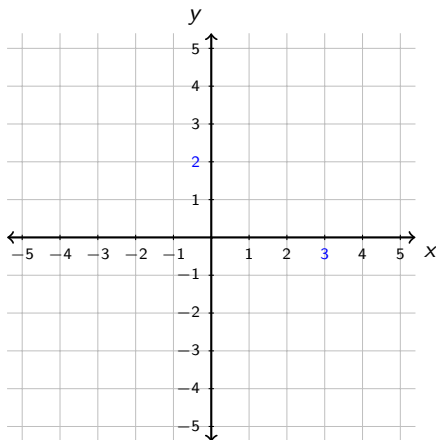
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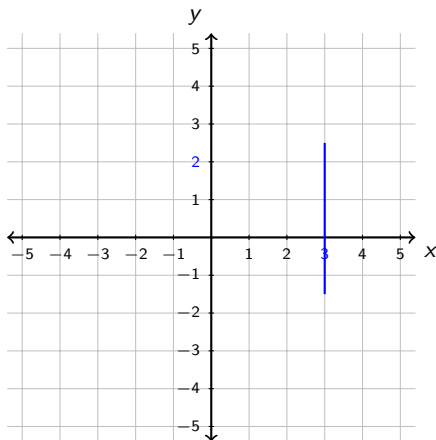
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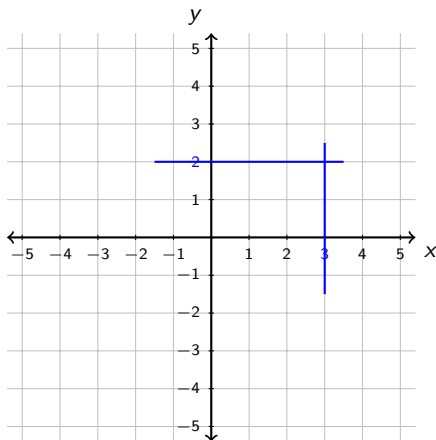
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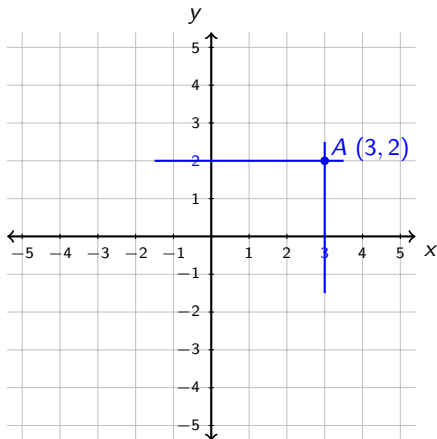
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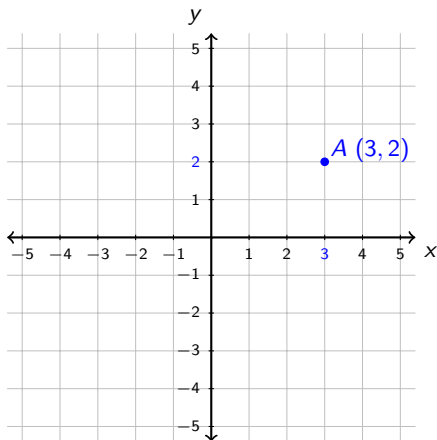




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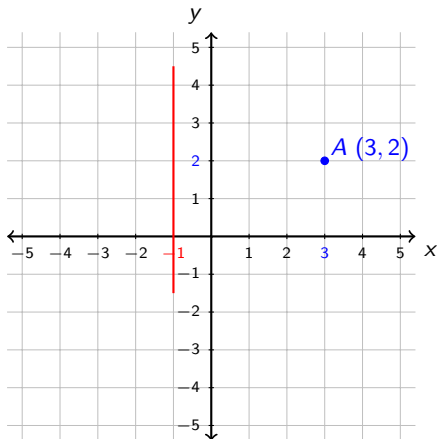
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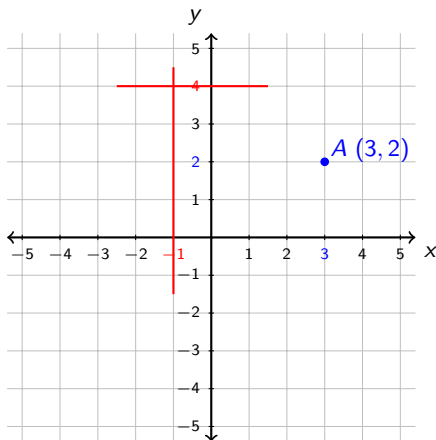
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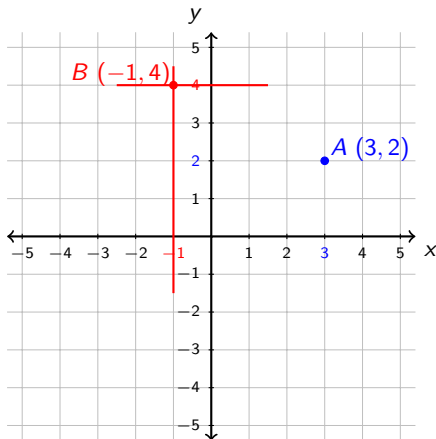
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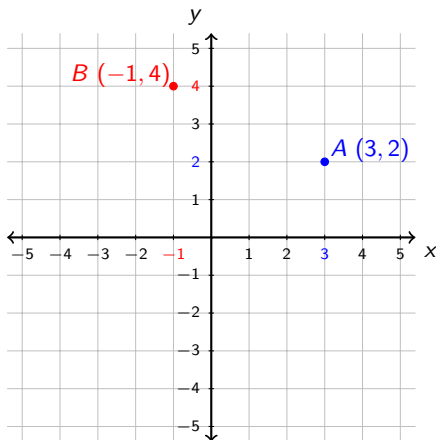
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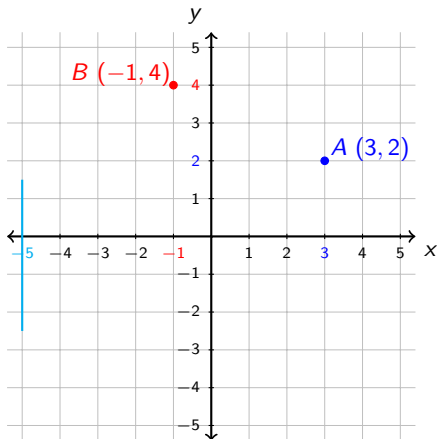
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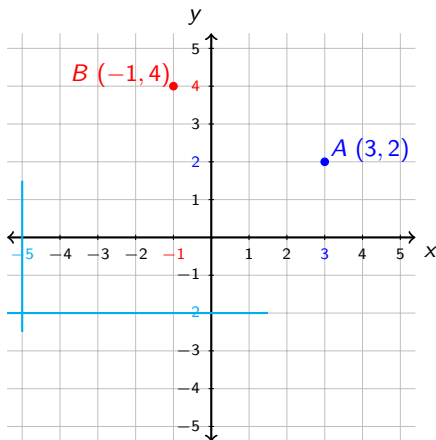
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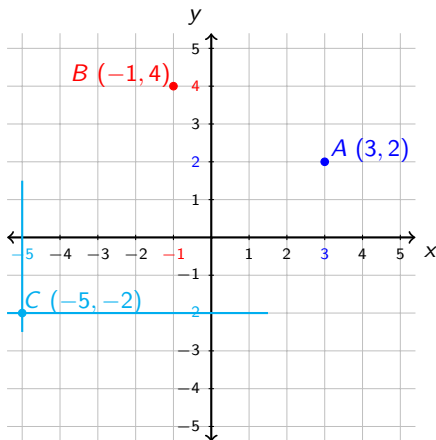
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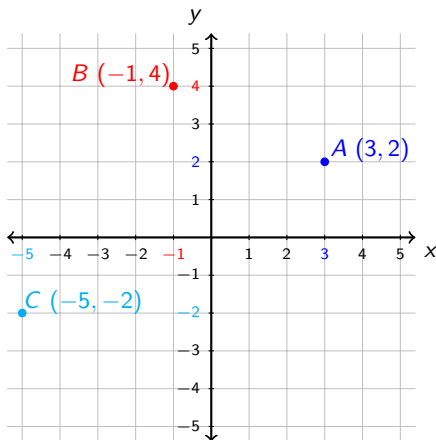




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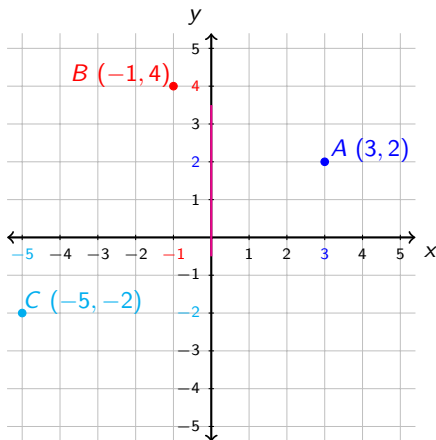
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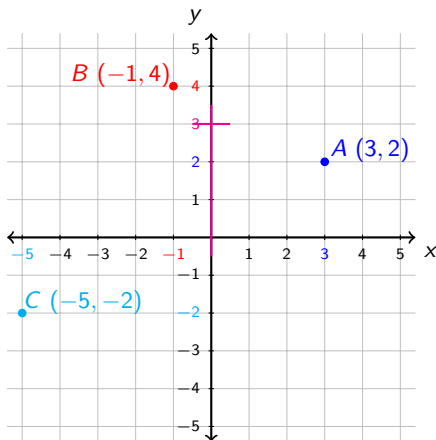
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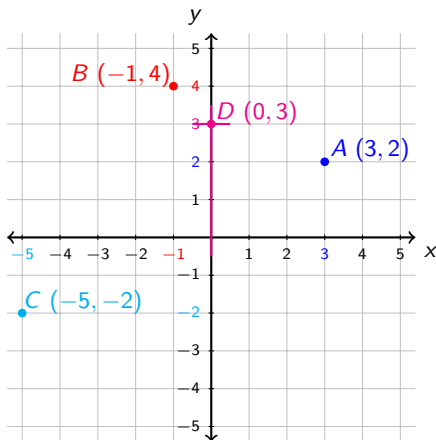
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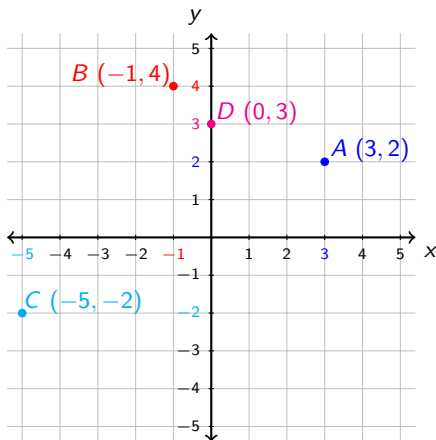
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So,  $y$  and  $x$  are **related to each other** in some way.



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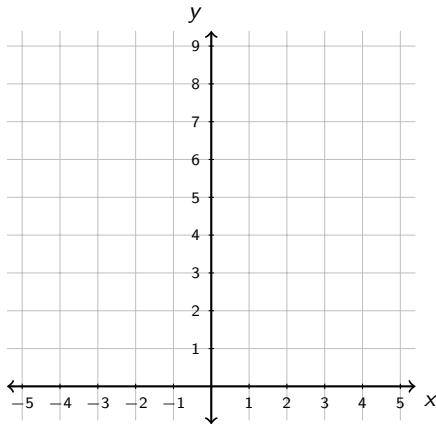
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By feeding values through our relationship, we obtain **coordinates** which we can display in the Cartesian plane!

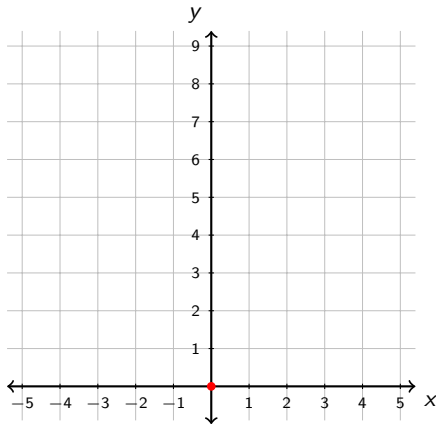
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Graph the coordinates:  $(0, 0)$ ,  $(1, 1)$ ,  $(2, 4)$ ,  $(3, 9)$



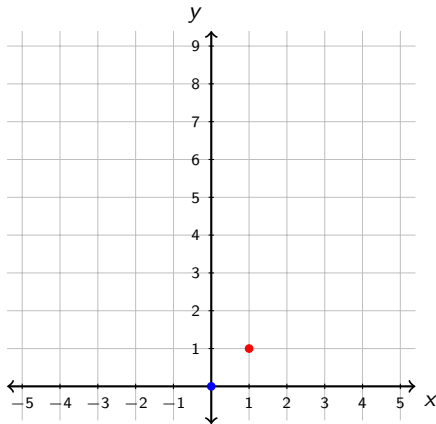
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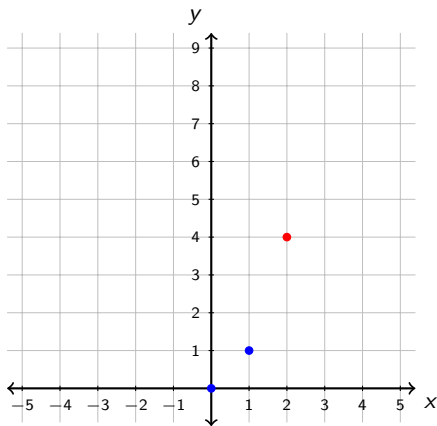
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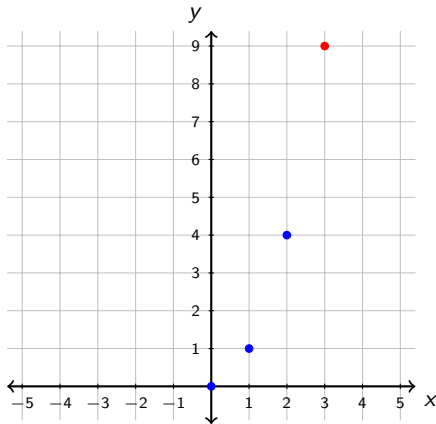
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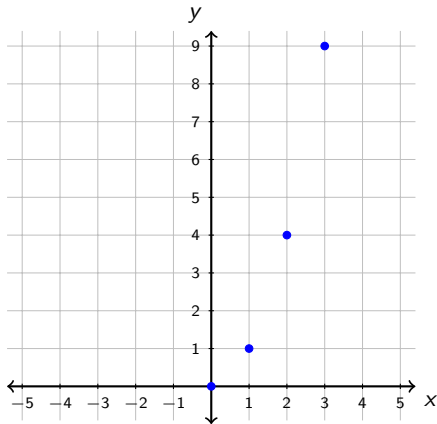
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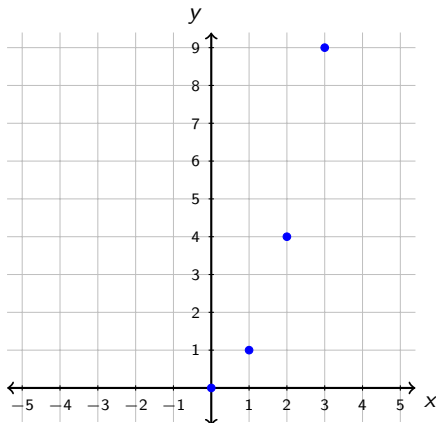
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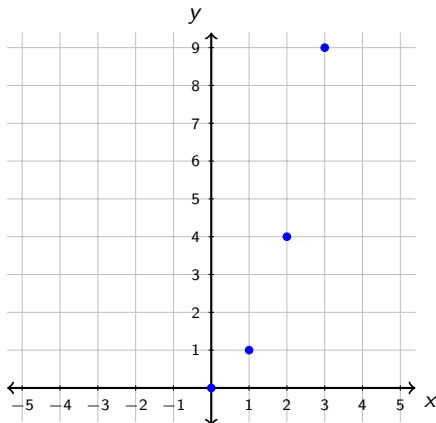
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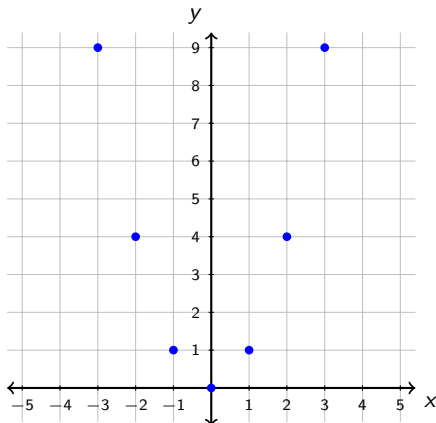
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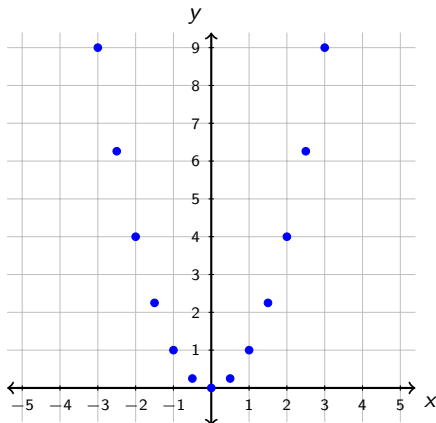
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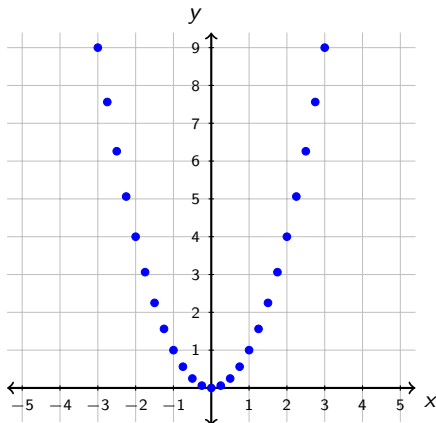
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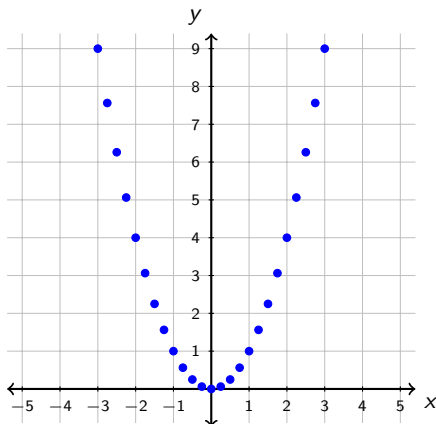
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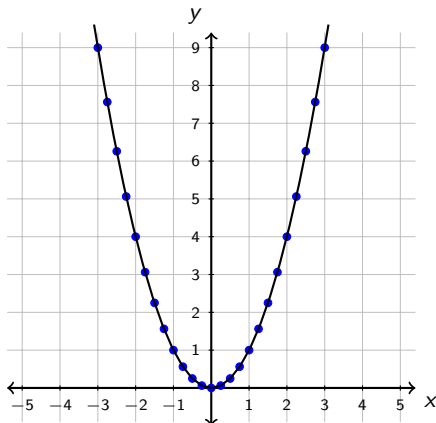


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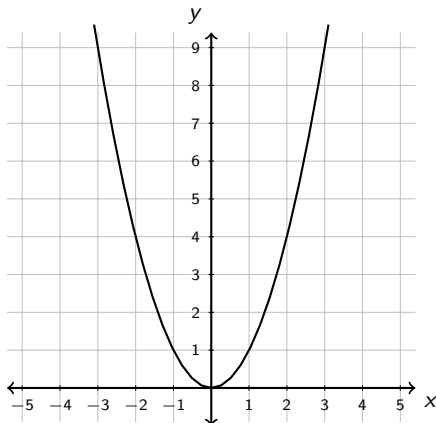


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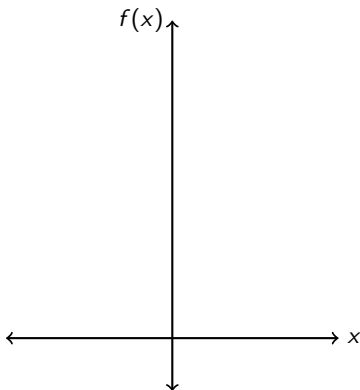
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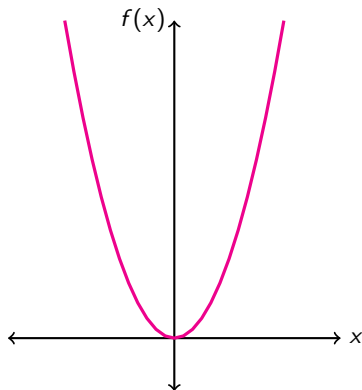
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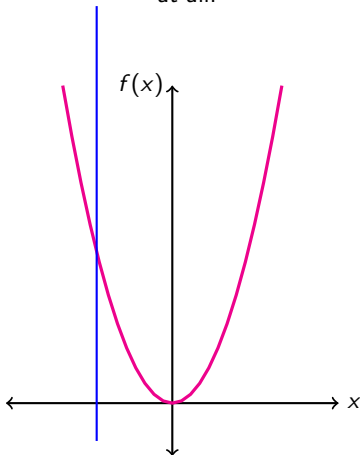
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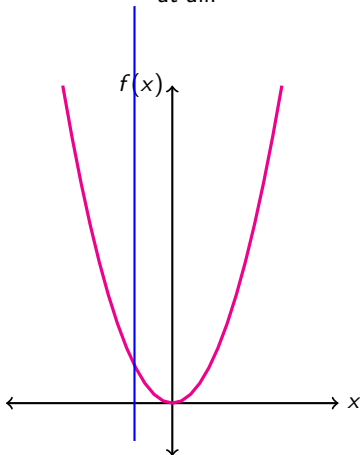
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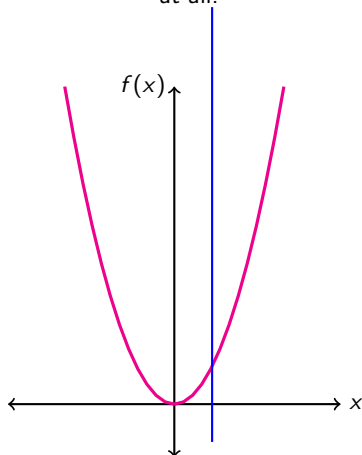
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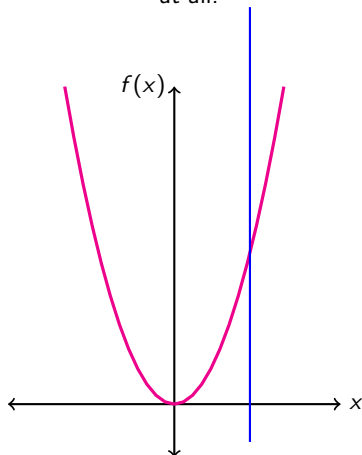
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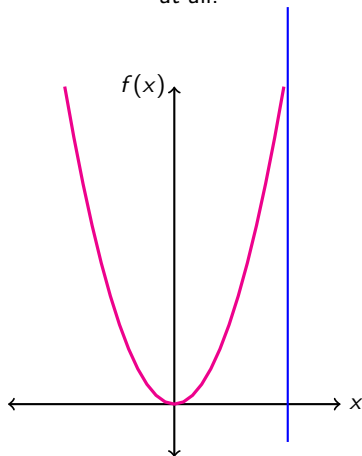
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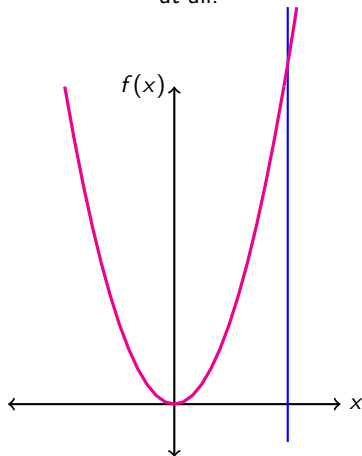
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**Two** outputs for the input 4, so **not** a function.

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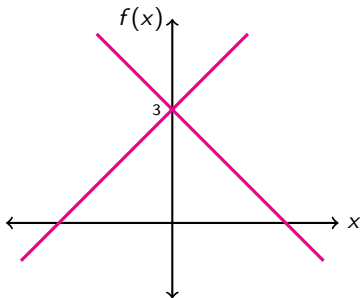
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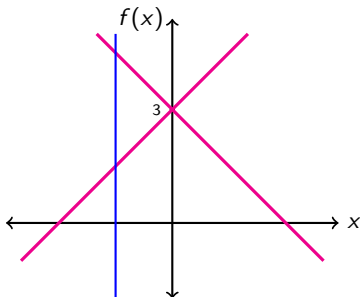


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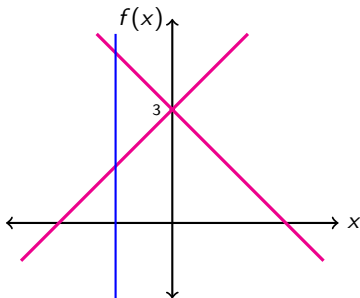


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Does not pass the vertical line test.

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The  $y$  notation is friendlier to rearrange.

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This is the equation of a straight line - which will clearly pass the vertical line test.



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**No**, because if we re-arrange it, we get  $y = \pm\sqrt{9 - x^2}$ . One value of  $x$  gives two values of  $y$ !

For example, if  $x = 0$ ,  $y = \pm 3$  (which should be obvious from the original equation).

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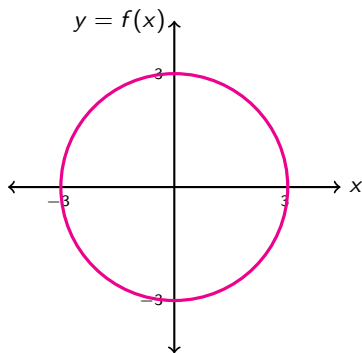
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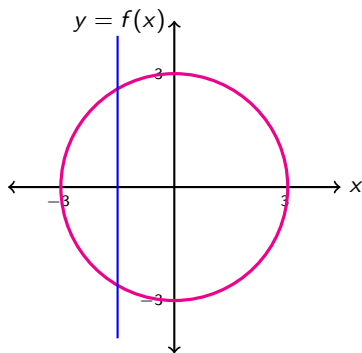
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- $f(z) = z + \frac{1}{z}$

## Using STUDYSmarter Resources

This resource was developed for UWA students by the *STUDYSmarter* team for the numeracy program. When using our resources, please retain them in their original form with both the *STUDYSmarter* heading and the UWA crest.



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