# **Please Note**

These pdf slides are configured for viewing on a computer screen.

Viewing them on hand-held devices may be difficult as they require a "slideshow" mode.

Do not try to print them out as there are many more pages than the number of slides listed at the bottom right of each screen.

Apologies for any inconvenience.

Linear and quadratic graphs Numeracy Workshop

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## Introduction

These slides introduce the cartesian plane and the features of linear and quadratic graphs.

Drop-in Study Sessions: Monday, Wednesday, Thursday, 10am-12pm, Meeting Room 2204, Second Floor, Social Sciences South Building, every week.

Website: Slides, worksheet, solutions.

www.studysmarter.uwa.edu.au  $\rightarrow$  Numeracy and Maths  $\rightarrow$  Online Resources

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Example: Start at the origin. Then, walk three metres East and five metres North

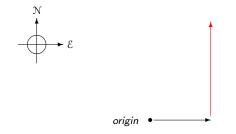


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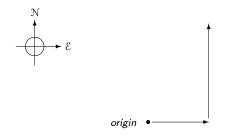


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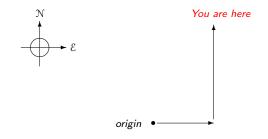


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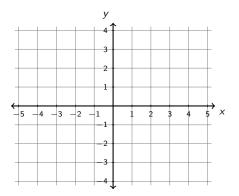
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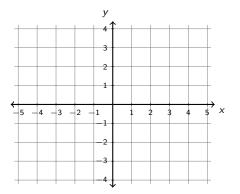
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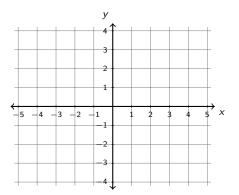
The most commonly used two-dimensional coordinate system is the Cartesian plane.



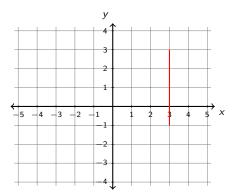
We specify points in the Cartesian plane as an ordered pair (x, y).



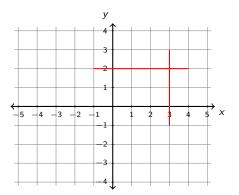
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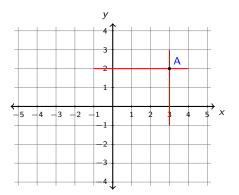
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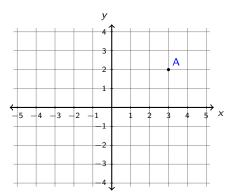
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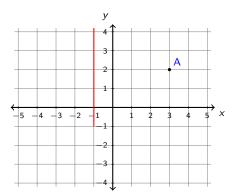
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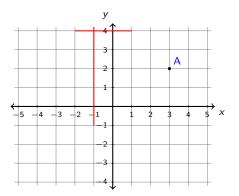
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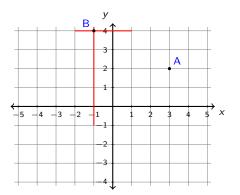
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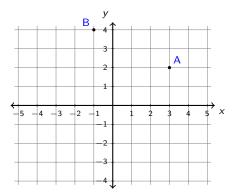
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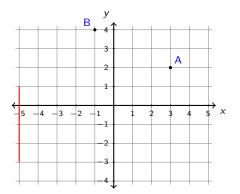
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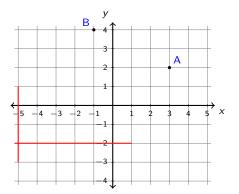
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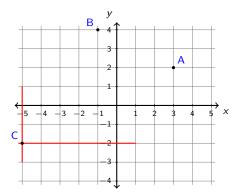
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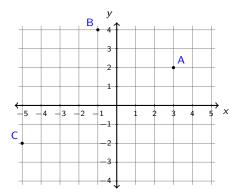
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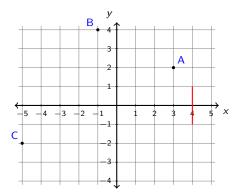
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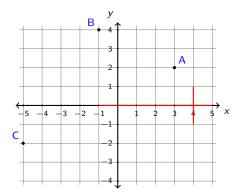
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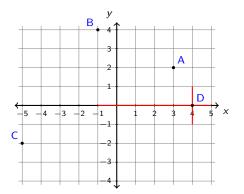
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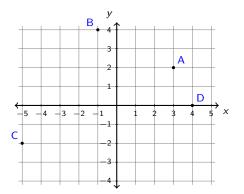
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# Equations and Relationships

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The above equation says, that whatever number x is, y is always the square of this number.

So, y and x are **fixed to each other** in some way.

 $y = x^2$ 

It is convenient sometimes to plot all the points (x, y) which satisfy an equation.

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Usually, this is done by feeding in x values and retrieving y values.

*x* = **0** 

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$$x = 0 \Rightarrow y = 0^2 = 0$$

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$$x = 1 \quad \Rightarrow \quad y = 1^2 = 1$$

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$$x = 1 \quad \Rightarrow \quad y = 1^2 = 1 \quad \rightarrow \quad (x, y) = (1, 1)$$

$$x = 2 \quad \Rightarrow \quad y = 2^2 = 4 \quad \rightarrow \quad (x, y) = (2, 4)$$

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$$x = 3 \quad \Rightarrow \quad y = 3^2 = 9 \quad \rightarrow \quad (x, y) = (3, 9)$$

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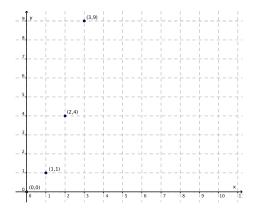
$$x = 3 \Rightarrow y = 3^2 = 9 \rightarrow (x, y) = (3, 9)$$

By feeding values through our relationship, we obtain points which we can display in the Cartesian plane!

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Graph the points:

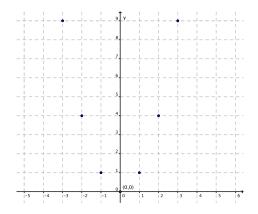
 $\{(0,0),(1,1),(2,4),(3,9)\}$ 

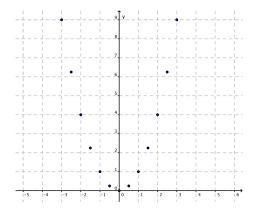


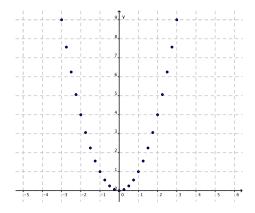
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The relationship becomes smooth as we plot more and more values.



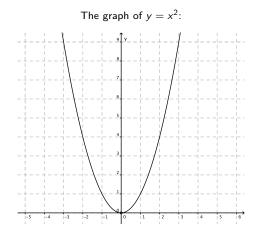




To get a smooth curve, we need to plot infinitely many points!

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This smooth curve is called the graph of the function.



We have seen that mathematical relationships have a shape when plotted on the Cartesian plane.

A graph is useful, because it represents the function completely.

A mathematicial relationship of the form

y = mx + c

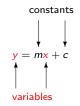
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#### A mathematicial relationship of the form



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y = 2x + 1

.

.

$$y = 2x + 1$$

.

.

Here, m = 2

y = 2x + 1

Here, m = 2 and c = 1.

.

$$y = 2x + 1$$

Here, 
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Plot some points (x, y) which satisfy the equation.

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$$x = 1 \quad \Rightarrow \quad y = 2 \times 1 + 1 = 3$$

$$y = 2x + 1$$

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$$x = 0 \Rightarrow y = 2 \times 0 + 1 = 1 \rightarrow (x, y) = (0, 1)$$

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$$x = 2 \Rightarrow y = 2 \times 2 + 1 = 5 \rightarrow (x, y) = (2, 5)$$

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 $x = -3 \Rightarrow y = 2 \times -3 + 1 = -5 \rightarrow (x, y) = (-3, -5)$ 

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Now plot these points on the Cartesian plane.

y -6 1 5 4 3 2 1  $\leftarrow -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1$ ÷x 2 ፍ -1-2 -3 -4 -5 -6

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у -6 1 5 4 3 2 1  $\leftarrow -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1$ ÷x 2 ፍ -1-2 -3 -4 -5 -6

у -61 5 4 3 2 1 ÷x -6 -5 -4 -3 -2 -12 ፍ -1-2 -3 -4 -5 -6

у 61 5 4 3 2 1 ÷x -6 -5 -4 -3 -2 -12 ፍ -1-2 -3 -4 -5 -6

y -6 1 5 4 3 2 -6 -5 -4 -3 -2₩ 6 × ŧ¥ 2 ፍ \_ 2 -3 -4 -5 -6



y -6 1 5 4 3 2 1 -6 -5 -4 -3 -2₩ 6 × ¥ 2 ፍ \_ 2 -3 -4 -5

-6

y = 2x + 1у -61 5 4 3 2 1 ← ; x -6 -5 -4 -3 -2 ¥ 2 2 -3 -4 -5 -6

y = 2x + 1у -61 5 4 3 2 1 ← ; x -6 -5 -4 -3 -2 ¥ 2 ġ 2 -3 -4 -5 -6

y = 2x + 1у -61 5 4 3 2 1 ← ; x -6 -5 -4 -3 -2 ¥ 2 2 -3 -4 -5 -6

y = 2x + 1у -61 5 4 3 2 1 ← ; x -6 -5 -4 -3 -2 ¥ 2 2 -3 -4 -5 -6

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Every 1 unit step we take to the right ("run") results in an m unit "rise" (or fall) vertically.

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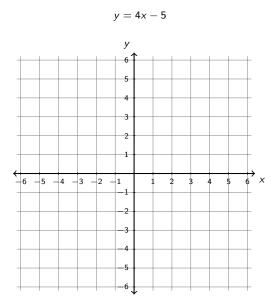
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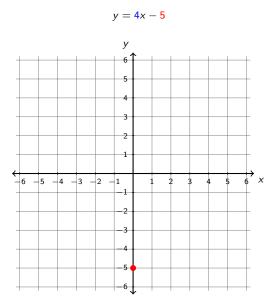
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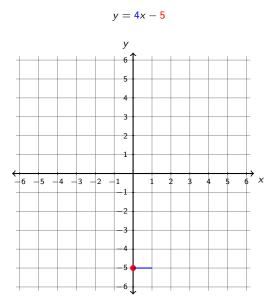
We call it the gradient of the line.

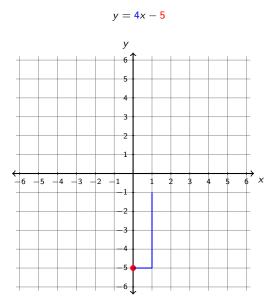


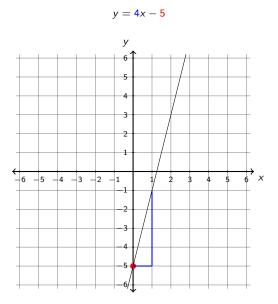
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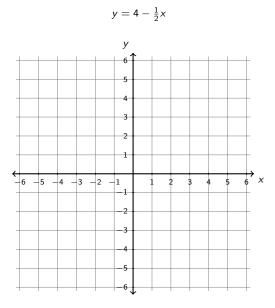
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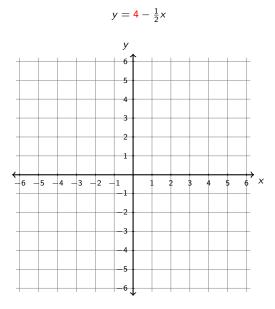


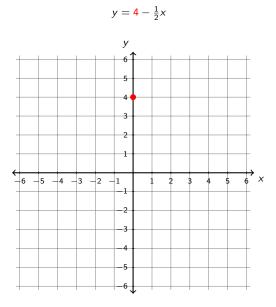


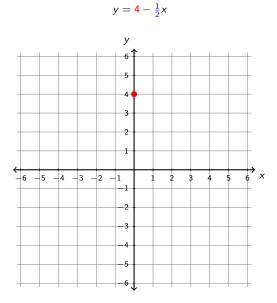


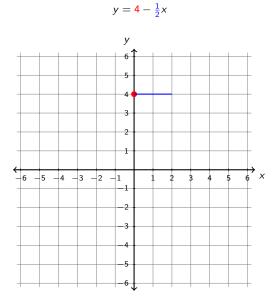


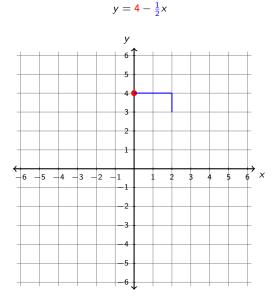


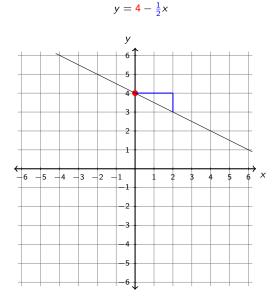












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Each of the following equations are linear functions:

$$5y = 3x - 20$$

$$2y + 8x = -3$$

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$$\frac{y}{x} = 4$$

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#### Have a go at rearranging them!

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#### **Quadratic Functions**

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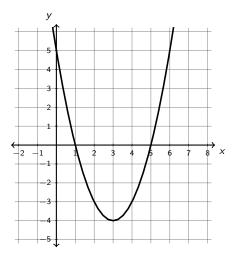
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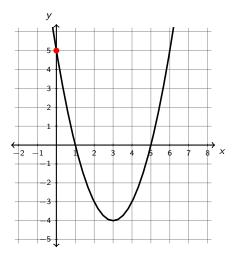
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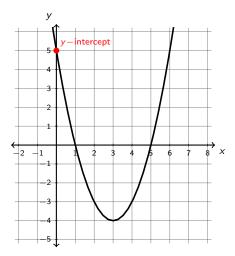
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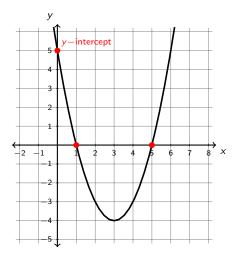
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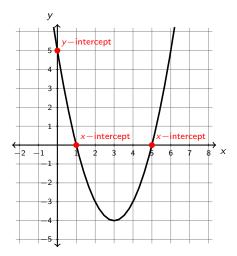
These functions take on the shape of a parabola when we graph them.

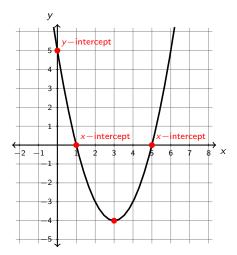


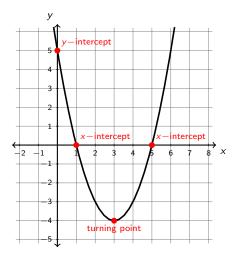


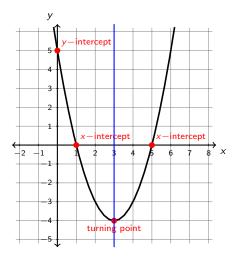


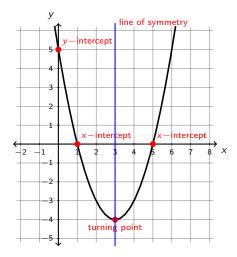












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Standard form: $y = x^2 - 6x + 5$ Factored form:y = (x - 1)(x - 5)Turning Point form: $y = (x - 3)^2 - 4$ 

Each form provides one or more properties of the quadratic graph.

Converting from one form to another requires algebraic manipulation which we won't discuss in this workshop.

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Equation smile/frown y-intercept line of symmetry

$$y = 2x^2 + 4x - 5 \qquad \text{smile}$$

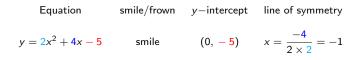
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Equation	smile/frown	y-intercept	line of symmetry
$y = 2x^2 + 4x - 5$	smile	(0, - <mark>5</mark> )	

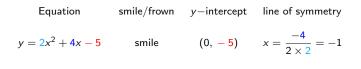
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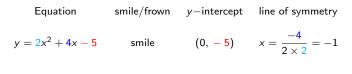
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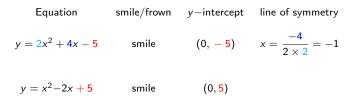
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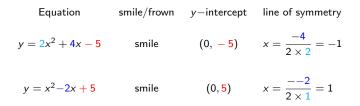


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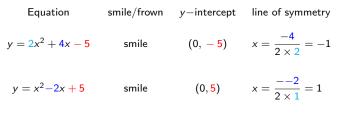


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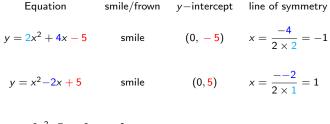
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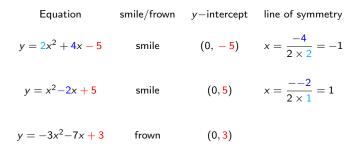
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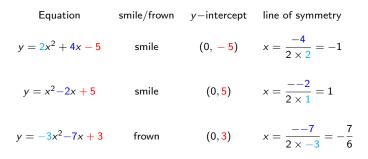


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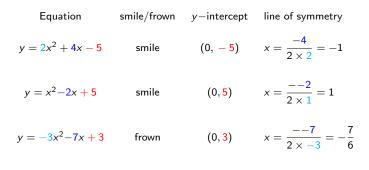


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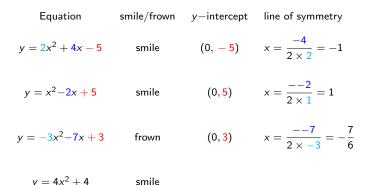
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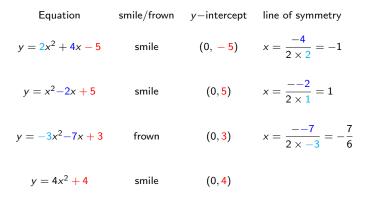


 $y = 4x^2 + 4$ 

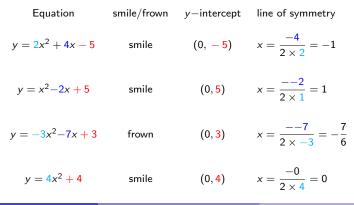
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Equation y-intercept line of symmetry  $y = 2x^2 + 4x - 5$  smile (0, -5) x = -1

Equation y-intercept line of symmetry  $y = 2x^2 + 4x - 5$  smile (0, -5) x = -1

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To find the y-coordinate of the turning point we simply substitute this x-value into the equation:

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$$y = 2(-1)^2 + 4(-1) - 5 = 2 - 4 - 5 = -7$$

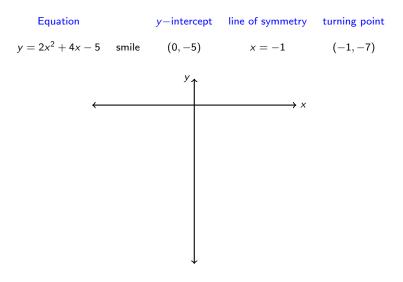
Equation y-intercept line of symmetry  $y = 2x^2 + 4x - 5$  smile (0, -5) x = -1

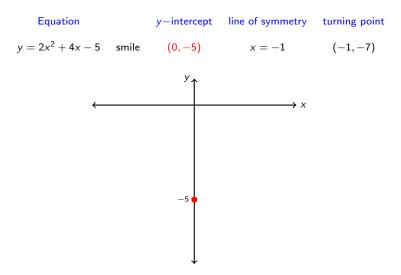
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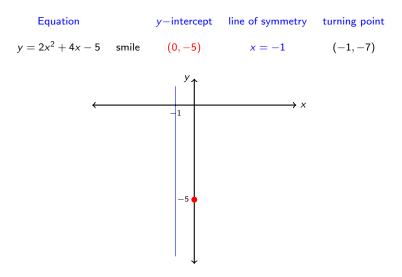
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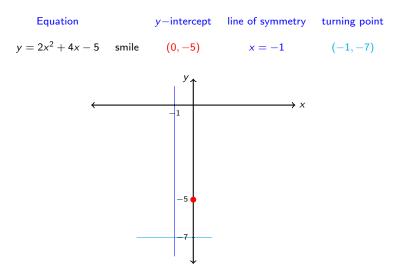
$$y = 2(-1)^2 + 4(-1) - 5 = 2 - 4 - 5 = -7$$

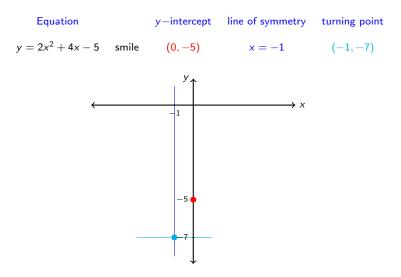
So, the turning point is (-1, -7).

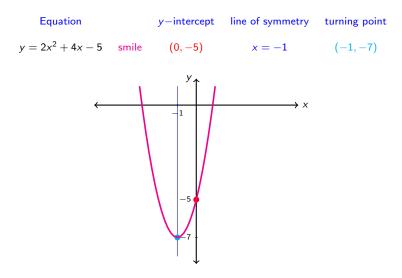


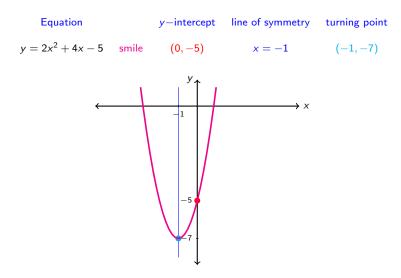




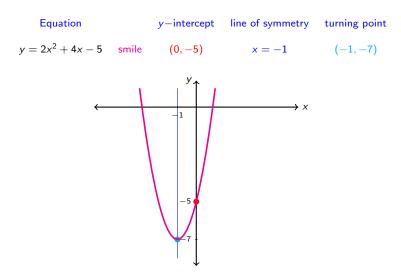




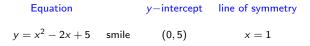




We now know that this quadratic has two x-intercepts.



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Equation y-intercept line of symmetry  $y = x^2 - 2x + 5$  smile (0,5) x = 1

To find the y-coordinate of the turning point we simply substitute the x-value into the equation:

Equation 
$$y$$
-intercept line of symmetry  
 $y = x^2 - 2x + 5$  smile  $(0,5)$   $x = 1$ 

To find the y-coordinate of the turning point we simply substitute the x-value into the equation:

$$y = (1)^2 - 2(1) + 5 =$$

Equation 
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 $y = x^2 - 2x + 5$  smile  $(0,5)$   $x = 1$ 

To find the y-coordinate of the turning point we simply substitute the x-value into the equation:

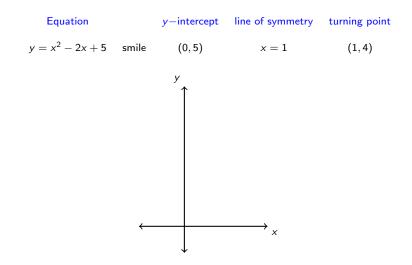
$$y = (1)^2 - 2(1) + 5 = 4$$

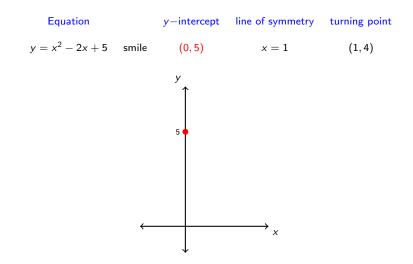
Equation y-intercept line of symmetry  $y = x^2 - 2x + 5$  smile (0,5) x = 1

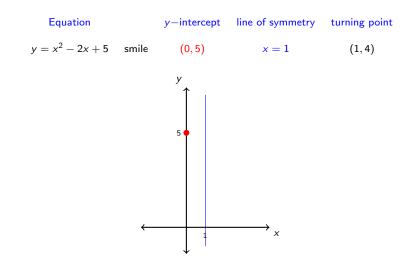
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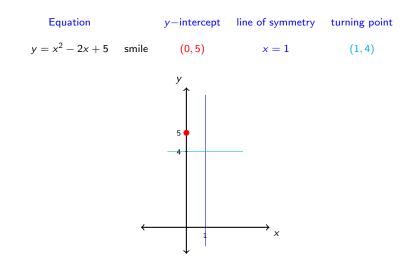
$$y = (1)^2 - 2(1) + 5 = 4$$

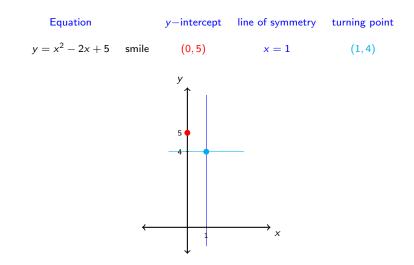
So, the turning point is (1, 4).



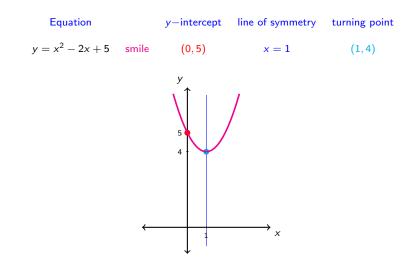




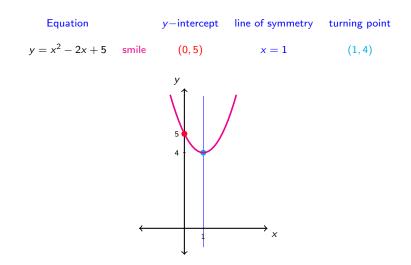




## Quadratic Functions: Standard Form



# Quadratic Functions: Standard Form



We can now see that this graph has no x-intercepts (which can also be discovered via the the quadratic formula).

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$$y = d(x - e)(x - f)$$

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In factored form, the sign of d determines whether it's a smile or frown

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In factored form, the sign of d determines whether it's a smile or frown, the x-intercepts are (e, 0) and (f, 0)

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In factored form, the sign of *d* determines whether it's a smile or frown, the *x*-intercepts are (e, 0) and (f, 0), which means that the line of symmetry is  $x = \frac{e+f}{2}$ .

Equation smile/frown *x*-intercepts line of symmetry

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Equation smile/frown x-intercepts line of symmetry

y = 2(x - 1)(x - 3) smile

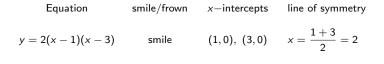
$$y = d(x - e)(x - f)$$

Equation	smile/frown	x-intercepts	line of symmetry
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$$y = d(x - e)(x - f)$$

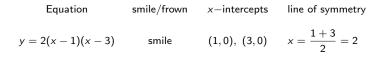
Equation	smile/frown	x-intercepts	line of symmetry
y=2(x-1)(x-3)	smile	(1,0), (3,0)	$x = \frac{1+3}{2} = 2$

$$y = d(x - e)(x - f)$$



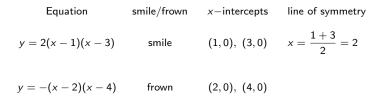
$$y = -(x-2)(x-4)$$

$$y = d(x - e)(x - f)$$

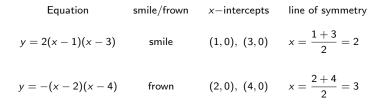


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 frown

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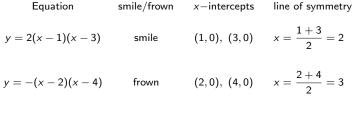


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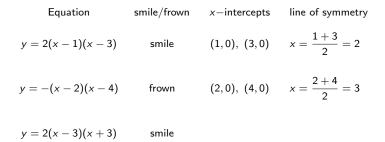
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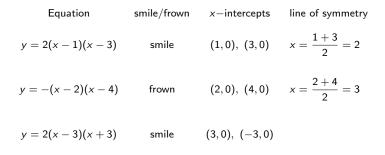


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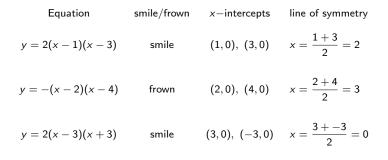
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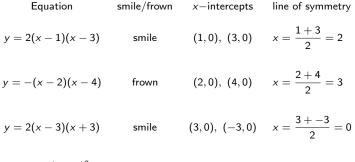


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 $y = -2(x+3)^2$ 

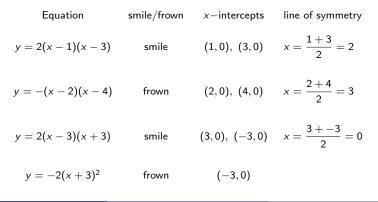
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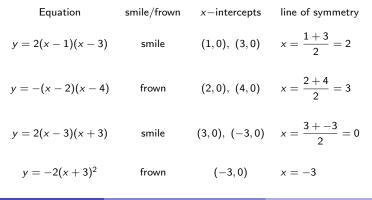
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y=2(x-1)(x-3)	smile	(1,0), (3,0)	$x = \frac{1+3}{2} = 2$
y=-(x-2)(x-4)	frown	(2,0), (4,0)	$x = \frac{2+4}{2} = 3$
y=2(x-3)(x+3)	smile	(3,0), (-3,0)	$x=\frac{3+-3}{2}=0$
$y = -2(x+3)^2$	frown		

Linear and quadratic graphs

$$y = d(x - e)(x - f)$$



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Equationx-interceptsline of symmetryy = -(x-2)(x-4)frown(2,0), (4,0)x = 3

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So, the *y*-intercept is (0, -8).



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As before, we have the x-value of the turning point because we have the line of symmetry.

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To find the y-coordinate of the turning point we simply substitute the x-value into the equation:

$$y = -(3-2)(3-4) = -(1)(-1) = 1$$

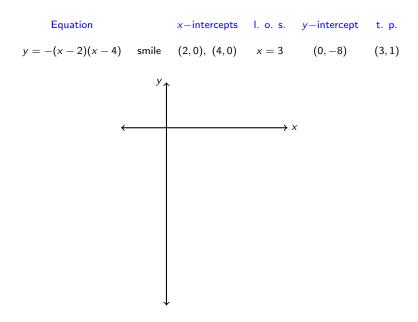
Equation x-intercepts line of symmetry y-intercept y = -(x - 2)(x - 4) frown (2,0), (4,0) x = 3 (0,-8)

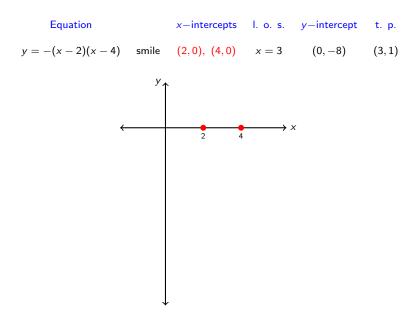
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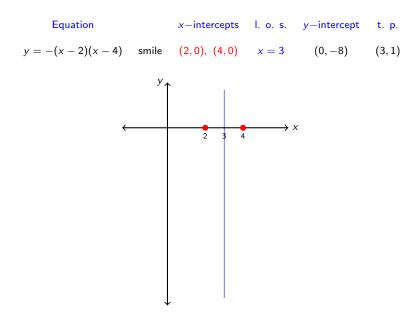
To find the y-coordinate of the turning point we simply substitute the x-value into the equation:

$$y = -(3-2)(3-4) = -(1)(-1) = 1$$

So, the turning point is (3, 1).







Equation x-intercepts I. o. s. y-intercept t. p. y = -(x-2)(x-4) smile (2,0), (4,0) x = 3 (0,-8) (3,1) У, → × 2

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Equation smile/frown turning point line of symmetry

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Equation smile/frown turning point line of symmetry  $y = 3(x-1)^2 + 4$ 

$$y = g(x-h)^2 + k$$

Equation	smile/frown	turning point	line of symmetry
$y = 3(x-1)^2 + 4$	smile		

$$y = g(x-h)^2 + k$$

Equation	smile/frown	turning point	line of symmetry
$y=3(x-1)^2+4$	smile	(1,4)	

$$y = g(x-h)^2 + k$$

Equation	smile/frown	turning point	line of symmetry
$y=3(x-1)^2+4$	smile	(1,4)	<i>x</i> = <b>1</b>

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In turning point form, the sign of g determines whether it's a smile or frown, the turning point is (h, k) and the line of symmetry is x = h.

Equation	smile/frown	turning point	line of symmetry
$y=3(x-1)^2+4$	smile	(1,4)	<i>x</i> = <b>1</b>

 $y = -2(x-2)^2 - 5$ 

$$y = g(x-h)^2 + k$$

In turning point form, the sign of g determines whether it's a smile or frown, the turning point is (h, k) and the line of symmetry is x = h.

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$y=3(x-1)^2+4$	smile	(1,4)	<i>x</i> = <b>1</b>

 $y = -2(x-2)^2 - 5$  frown

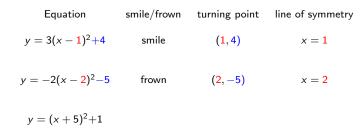
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Equation	smile/frown	turning point	line of symmetry
$y=3(x-1)^2+4$	smile	(1,4)	<i>x</i> = <b>1</b>
$y = -2(x-2)^2 - 5$	frown	(2, -5)	

$$y = g(x-h)^2 + k$$

Equation	smile/frown	turning point	line of symmetry
$y=3(x-1)^2+4$	smile	(1,4)	<i>x</i> = <b>1</b>
$y = -2(x - 2)^2 - 5$	frown	<b>(2</b> , -5)	x = 2

$$y = g(x-h)^2 + k$$



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Equation	smile/frown	turning point	line of symmetry
$y=3(x-1)^2+4$	smile	(1,4)	<i>x</i> = <b>1</b>
$y=-2(x-2)^2-5$	frown	(2, -5)	x = 2
$y = (x+5)^2 + 1$	smile		

$$y = g(x-h)^2 + k$$

Equation	smile/frown	turning point	line of symmetry
$y=3(x-1)^2+4$	smile	(1,4)	<i>x</i> = <b>1</b>
$y=-2(x-2)^2-5$	frown	(2, -5)	<i>x</i> = 2
$y = (x + 5)^2 + 1$	smile	(-5,1)	

$$y = g(x-h)^2 + k$$

Equation	smile/frown	turning point	line of symmetry
$y = 3(x-1)^2 + 4$	smile	(1,4)	<i>x</i> = <b>1</b>
$y=-2(x-2)^2-5$	frown	(2, -5)	x = 2
$y = (x + 5)^2 + 1$	smile	( - 5, 1)	x = -5

$$y = g(x-h)^2 + k$$

Equation	smile/frown	turning point	line of symmetry
$y=3(x-1)^2+4$	smile	(1,4)	<i>x</i> = <b>1</b>
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$$y = g(x-h)^2 + k$$

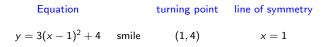
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$y = (x+5)^2 + 1$	smile	(-5,1)	x = -5
$y = -(x + 3)^2$	frown	( - 3, 0)	

$$y = g(x-h)^2 + k$$

Equation	smile/frown	turning point	line of symmetry
$y=3(x-1)^2+4$	smile	(1,4)	<i>x</i> = 1
$y=-2(x-2)^2-5$	frown	<b>(</b> 2, -5 <b>)</b>	x = 2
$y = (x+5)^2 + 1$	smile	(-5,1)	x = -5
$y = -(x + 3)^2$	frown	( - 3, 0)	x = -3



Equation turning point line of symmetry  $y = 3(x-1)^2 + 4$  smile (1,4) x = 1

The *y*-intercept isn't obvious in this form but we can find it by substituting x = 0 into the equation:

Equation turning point line of symmetry  $y = 3(x - 1)^2 + 4$  smile (1, 4) x = 1

The *y*-intercept isn't obvious in this form but we can find it by substituting x = 0 into the equation:

$$y = 3(0-1)^2 + 4$$

Equation turning point line of symmetry  $y = 3(x - 1)^2 + 4$  smile (1, 4) x = 1

The *y*-intercept isn't obvious in this form but we can find it by substituting x = 0 into the equation:

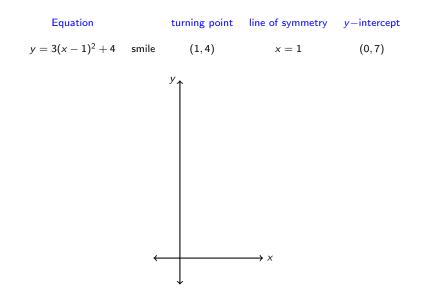
$$y = 3(0-1)^2 + 4 = 3(-1)^2 + 4 = 7$$

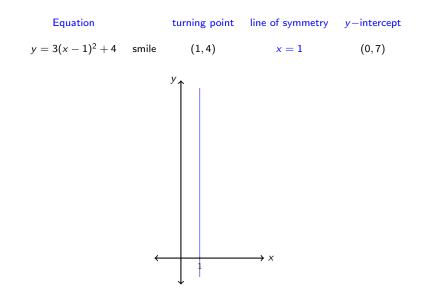
Equation turning point line of symmetry  $y = 3(x - 1)^2 + 4$  smile (1, 4) x = 1

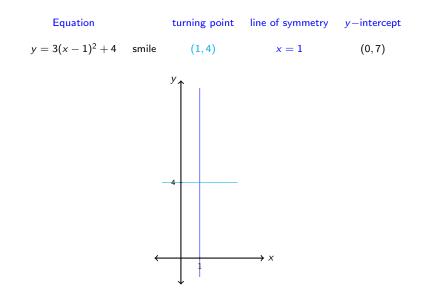
The *y*-intercept isn't obvious in this form but we can find it by substituting x = 0 into the equation:

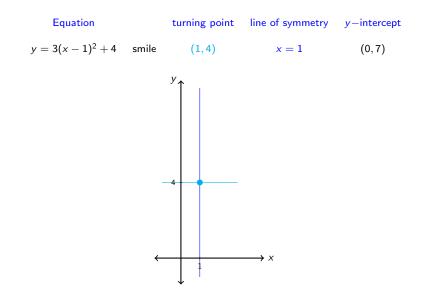
$$y = 3(0-1)^2 + 4 = 3(-1)^2 + 4 = 7$$

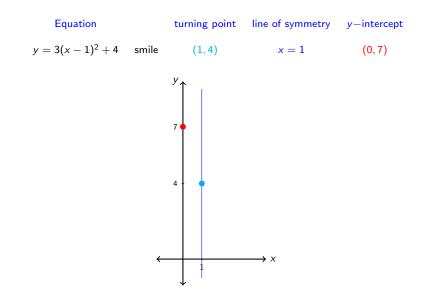
So, the *y*-intercept is (0, 7).

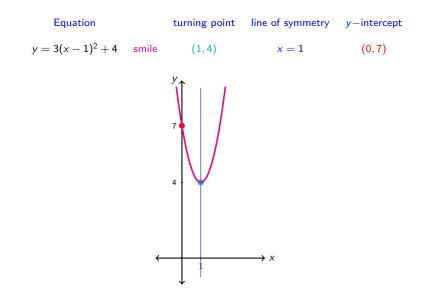












#### Using STUDYSmarter Resources

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