Please Note

These pdf slides are configured for viewing on a computer screen.

Viewing them on hand-held devices may be difficult as they require a "slideshow" mode.

Do not try to print them out as there are many more pages than the number of slides listed at the bottom right of each screen.

Apologies for any inconvenience.

Transformations of functions

Numeracy Program

geoff.coates@uwa.edu.au

STUDYSmarter
Learning, Language and Research Skills

Introduction

These slides introduce a useful way to graph functions based on manipulating (or **transforming**) the basic form of the graph.

Drop-in Study Sessions: Monday, Wednesday, Thursday, 10am-12pm, Meeting Room 2204, Second Floor, Social Sciences South Building, **every week.**

Website: Slides, notes, worksheets.

http://www.studysmarter.uwa.edu.au → Numeracy → Online Resources

Email: *geoff.coates@uwa.edu.au*

Workshops coming up

Week 8: Friday 26/4 (1-1.45pm): Fixing your maths mistakes

Week 9: Tuesday 30/4 (12-12.45pm): Introduction to calculus

Week 9: Thursday 2/5 (1-1.45pm): Calculating Limits (1 variable functions)

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The $y-$value of the turning point is then just $y = 3$.

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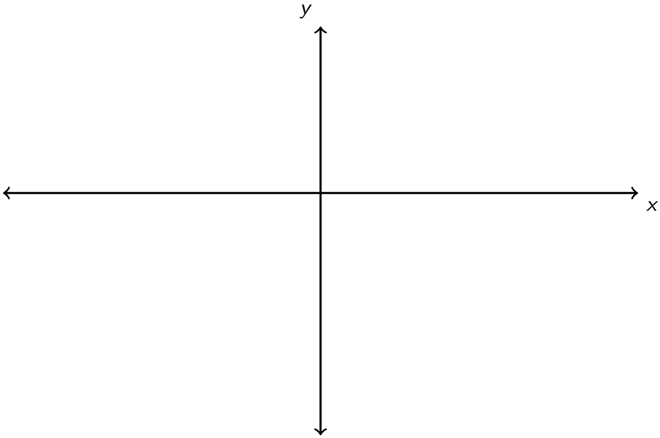

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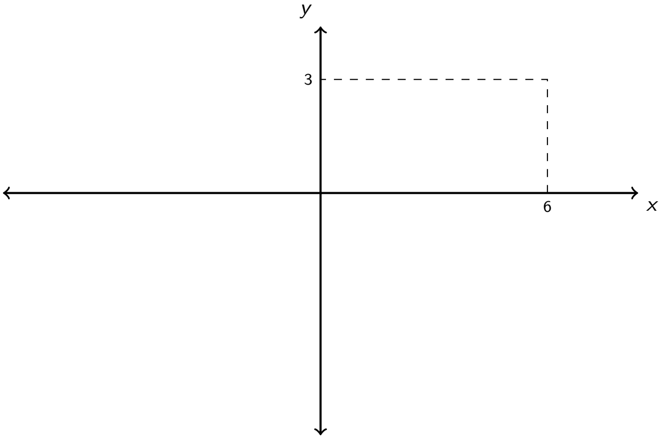

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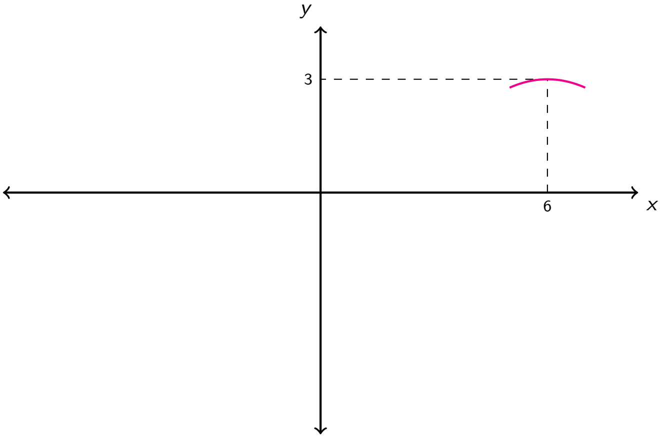

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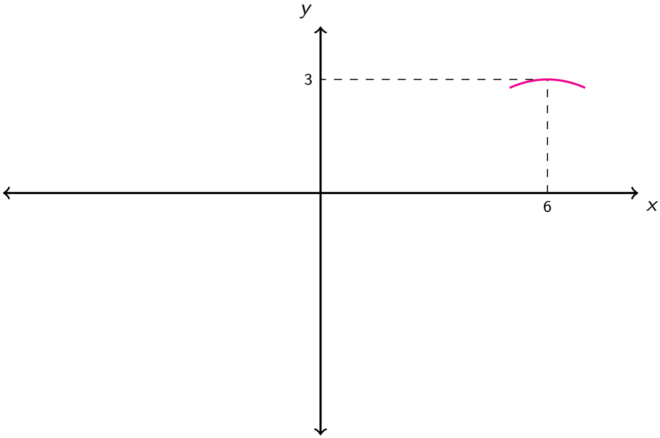

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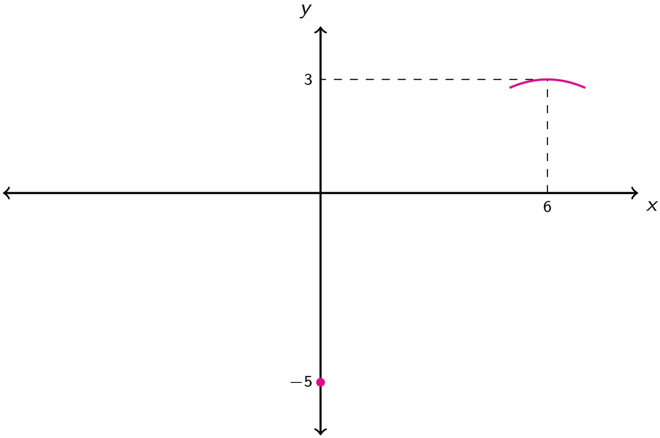

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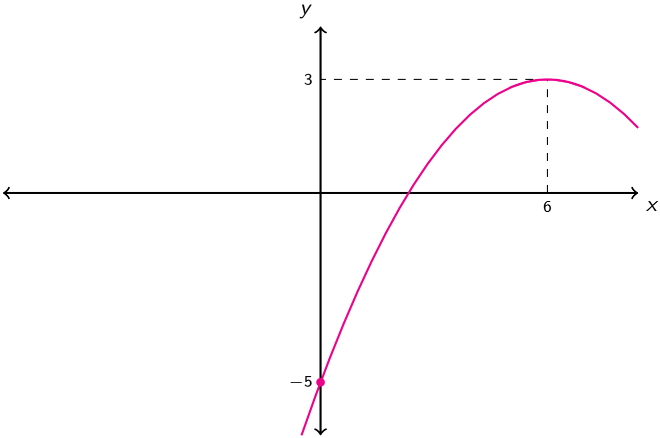

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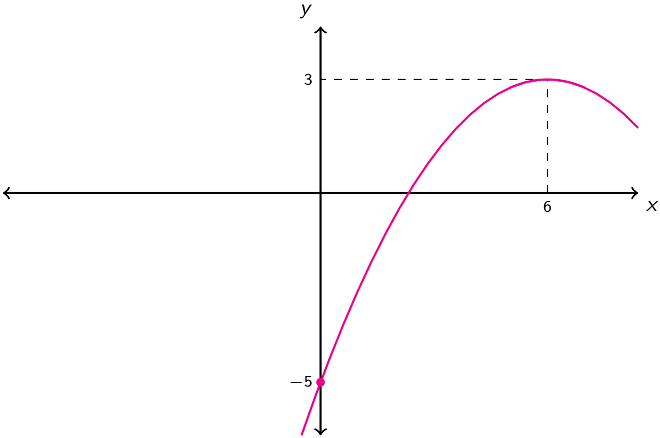

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Note: For finer details such as the $x-$intercepts, more calculations are needed.

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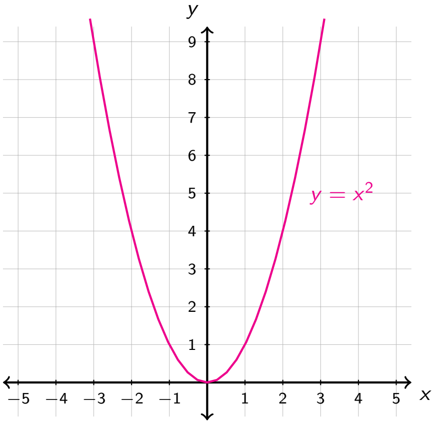

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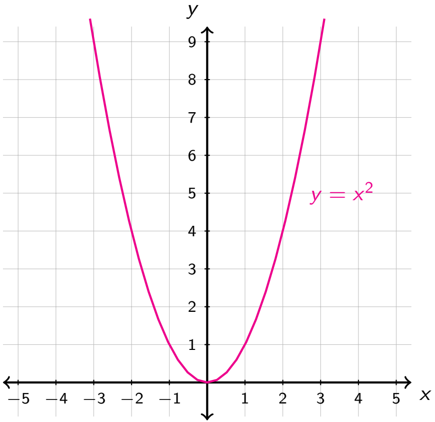

Note: I shall use graph paper for a while to make the functions clear.

Square Root Function

Here is another basic function. Consider the relationship where y is the square root of x.

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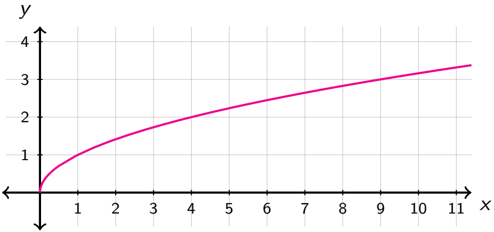

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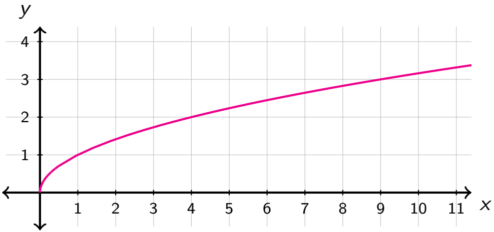

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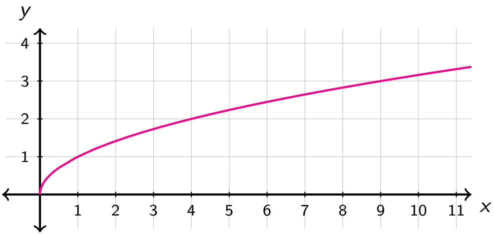

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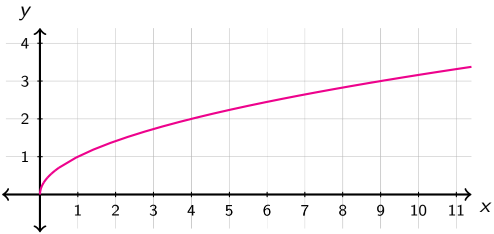

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Note: the square root function is half of the $y = x^2$ function lying on its side.

Reciprocal Function

Here is one more basic function. Consider the relationship where the product of x and y is 1.

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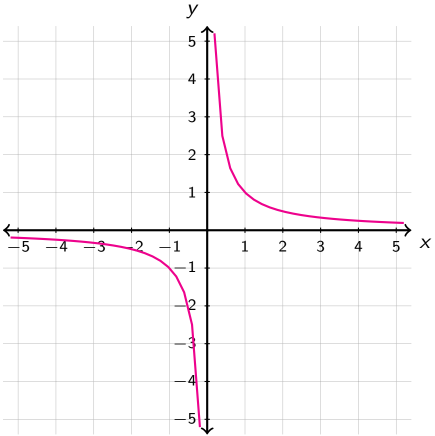

Transformations

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A translation is a shift, that is picking up the graph and moving it.

Vertical Translations

To translate a graph c units **upwards** (or downwards if c is negative)
we add c *onto the entire function*.

Vertical Translations

Example: $y = x^2 + 3$ is the same shape as $y = x^2$ but shifted three units upwards.

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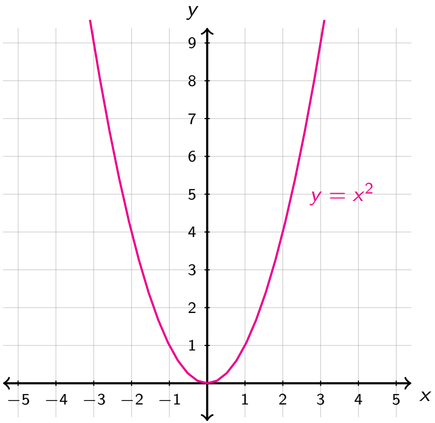

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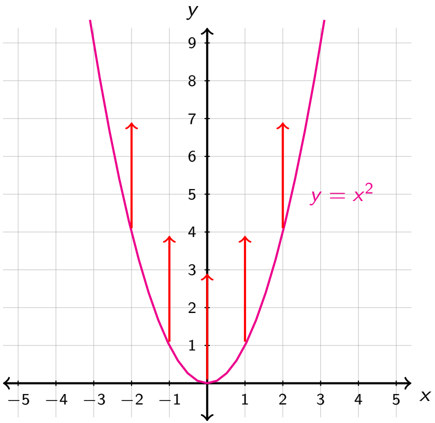

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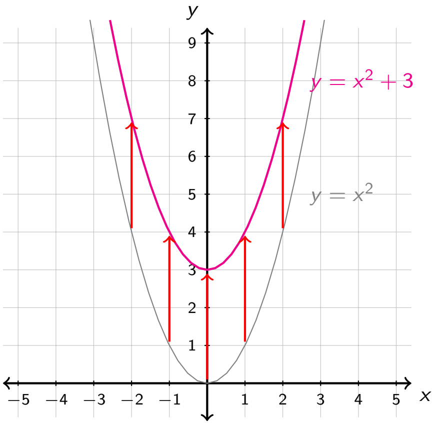

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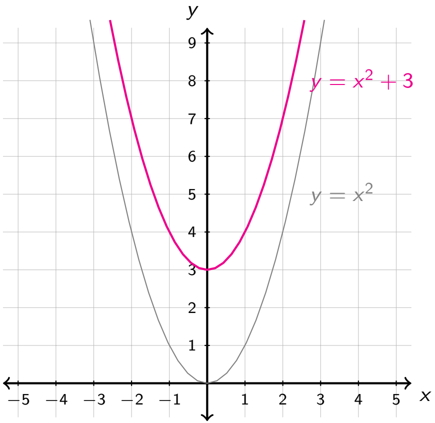

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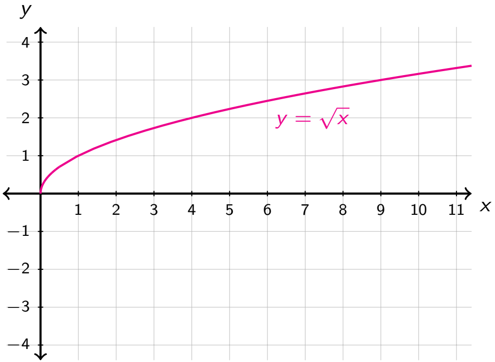

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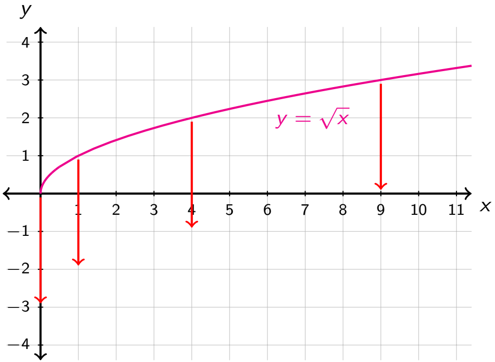

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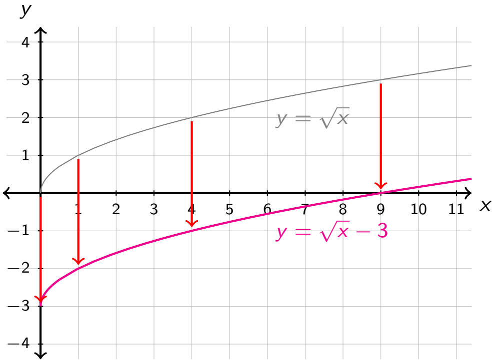

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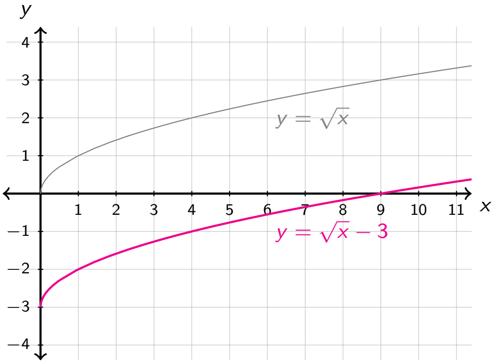

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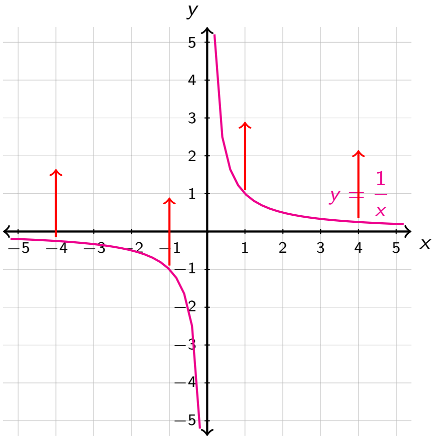

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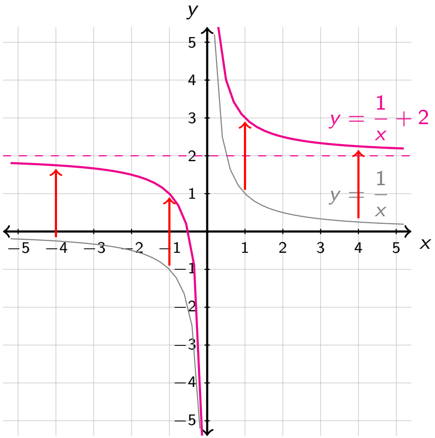

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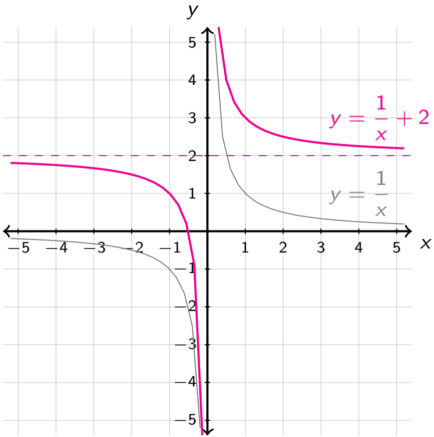

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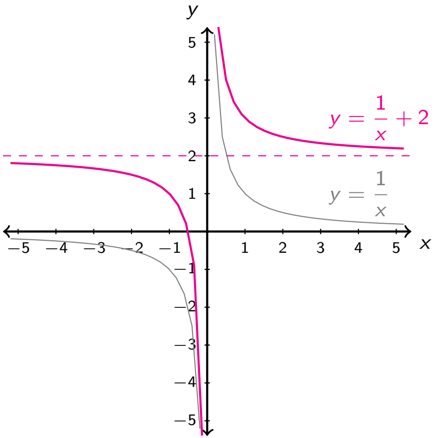

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To translate a graph c units to the left, we replace x with $x+c$ in the function.

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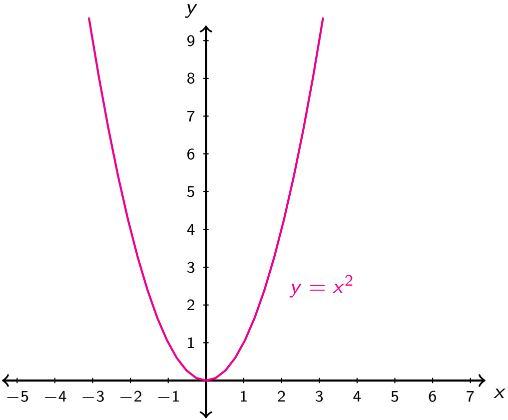

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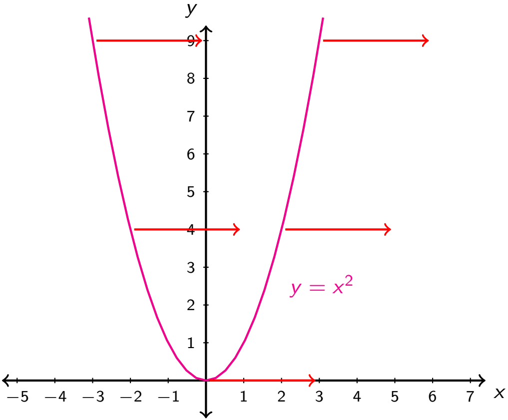

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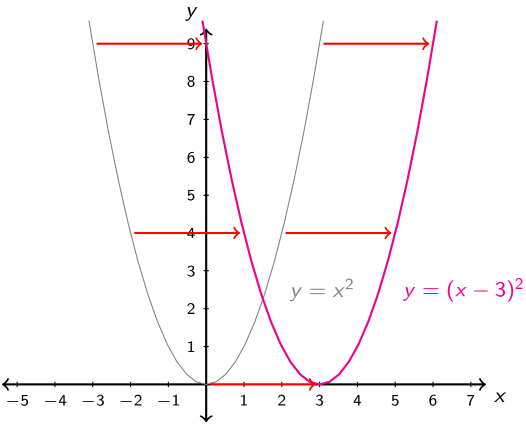

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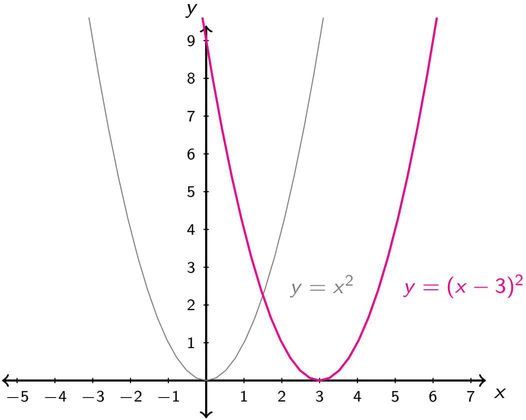

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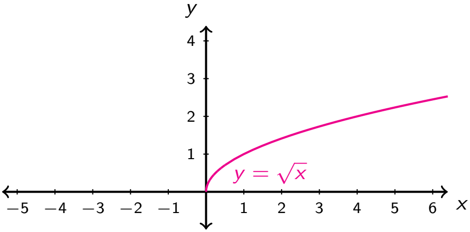

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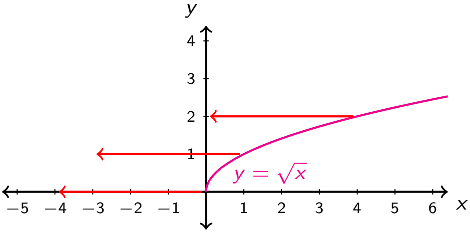

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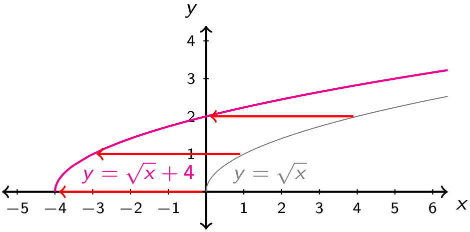

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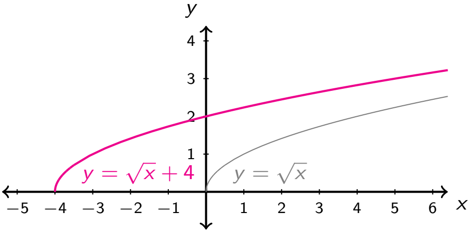

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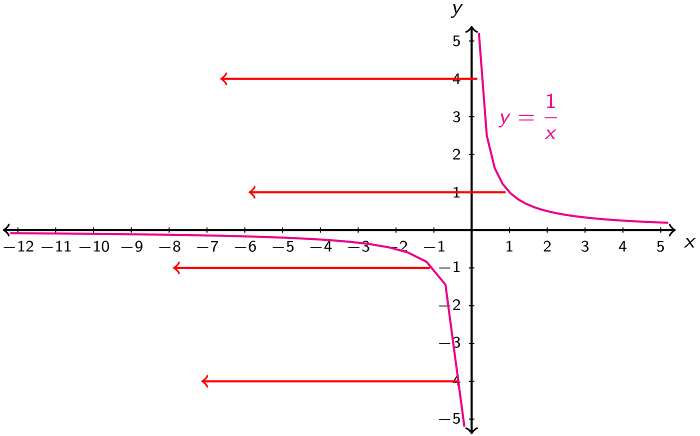

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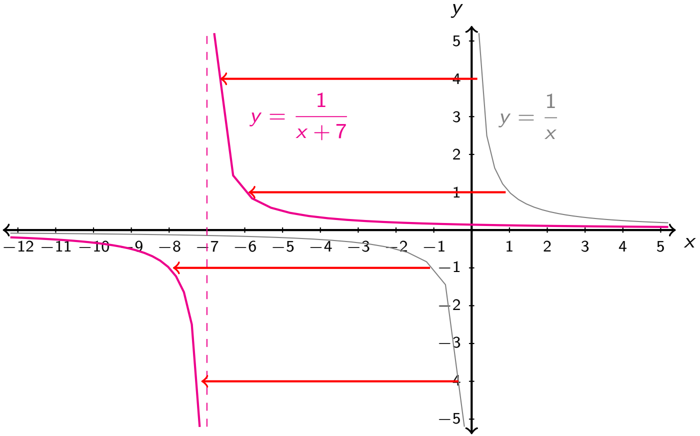

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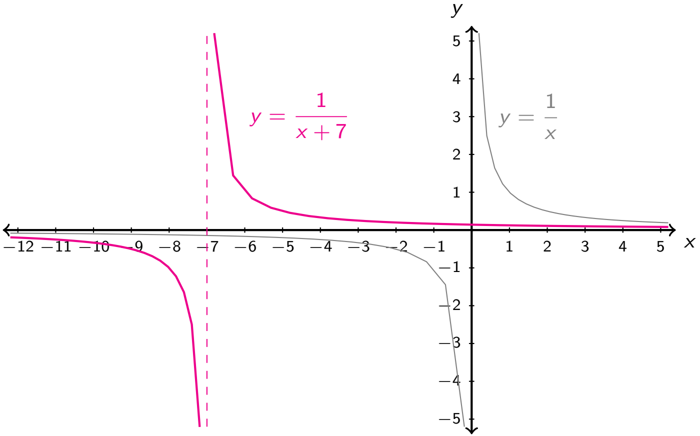

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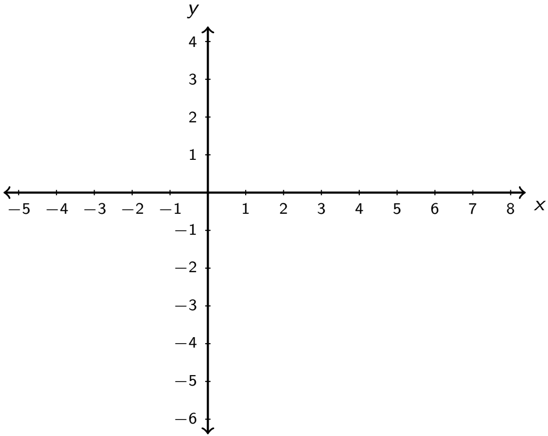

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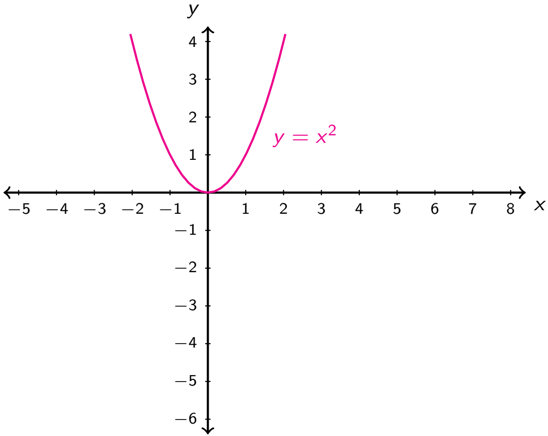

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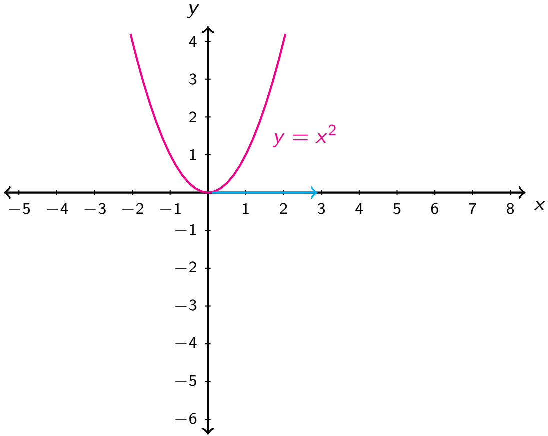

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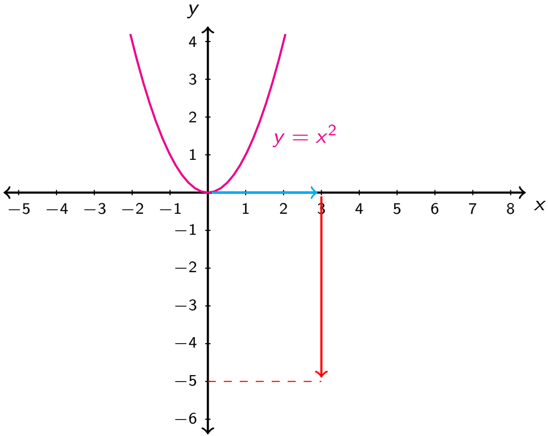

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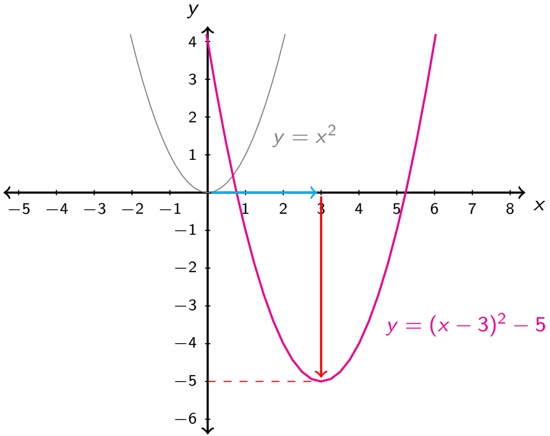

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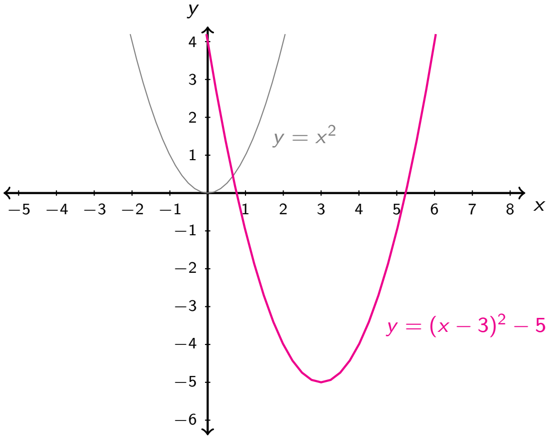

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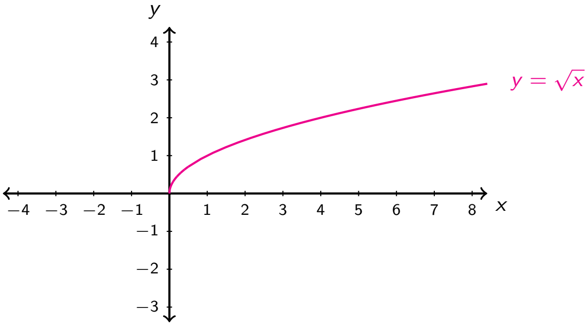

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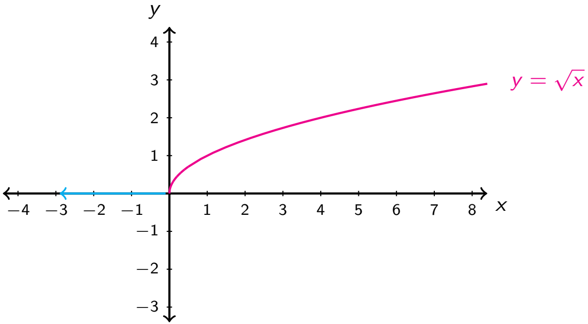

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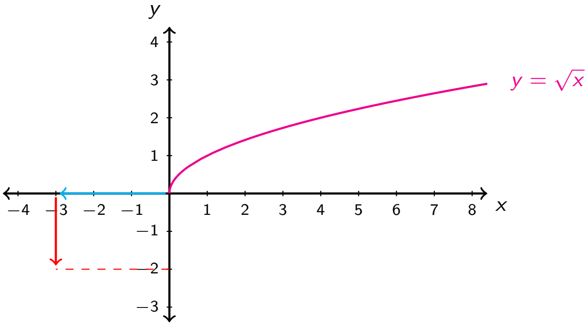

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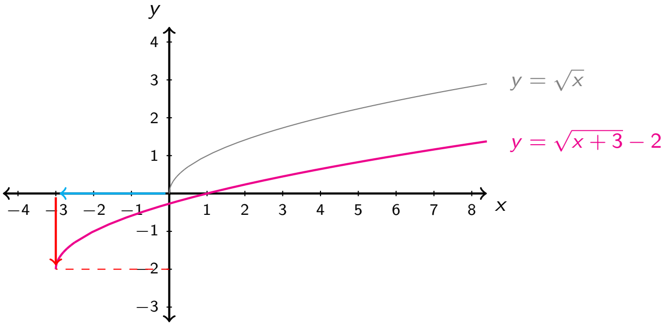

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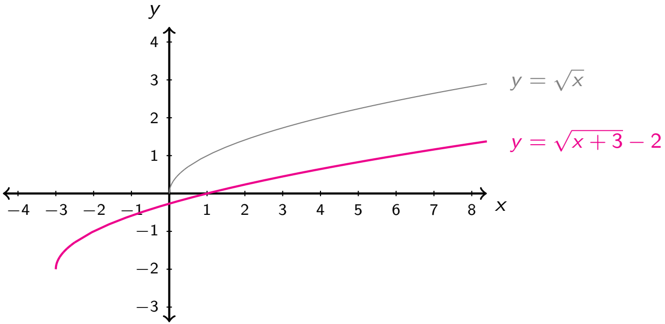

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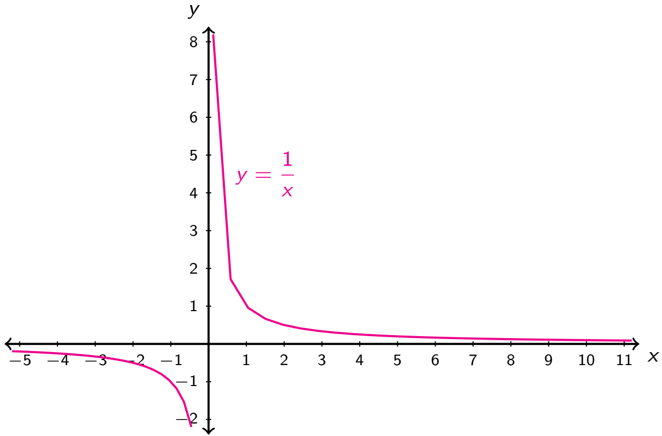

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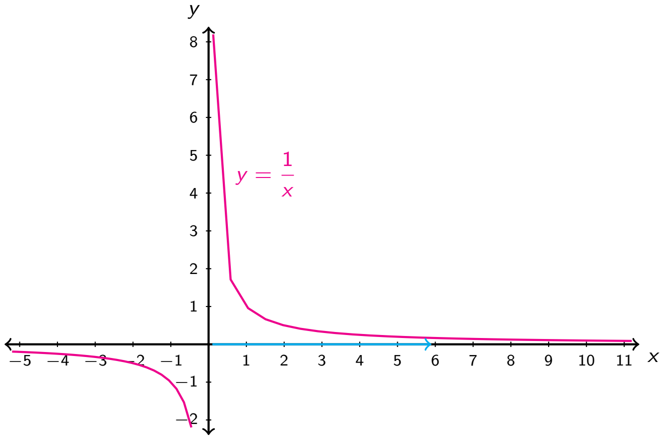

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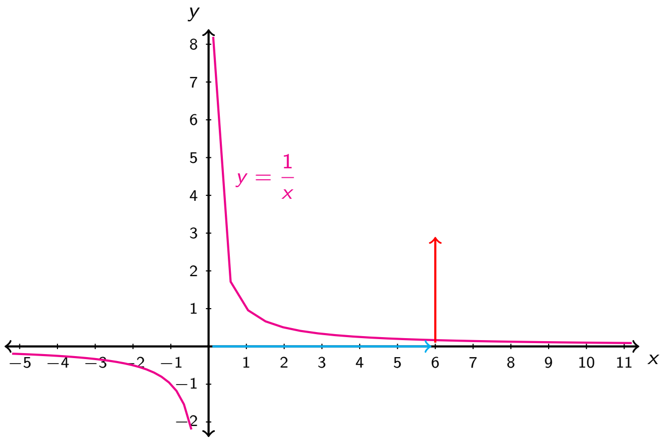

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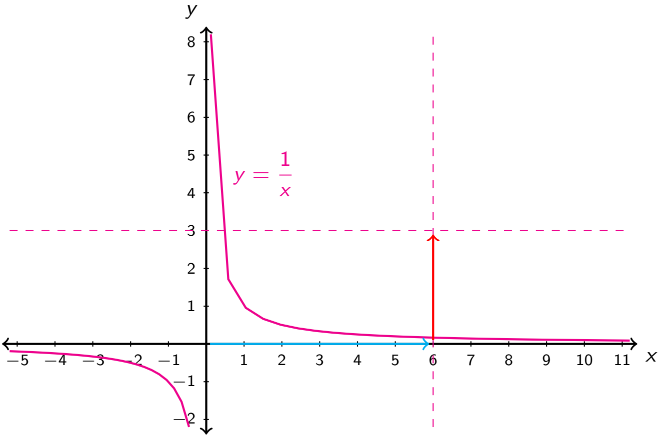

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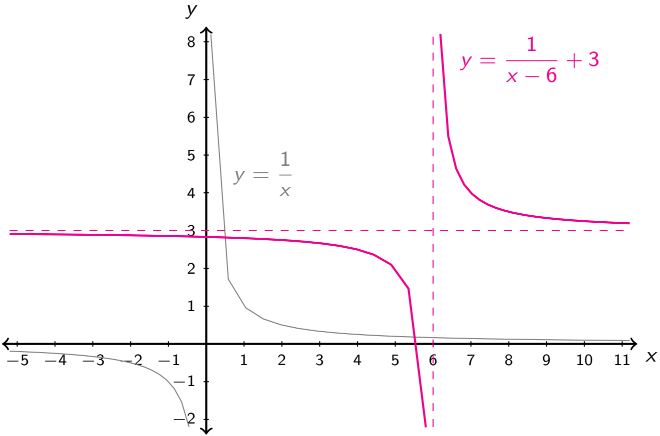

Dilations

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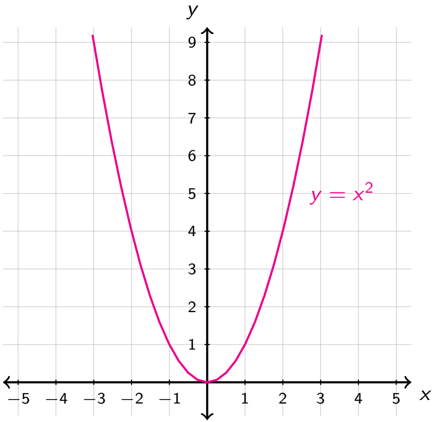

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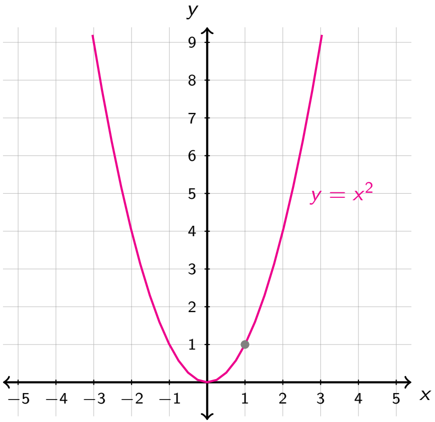

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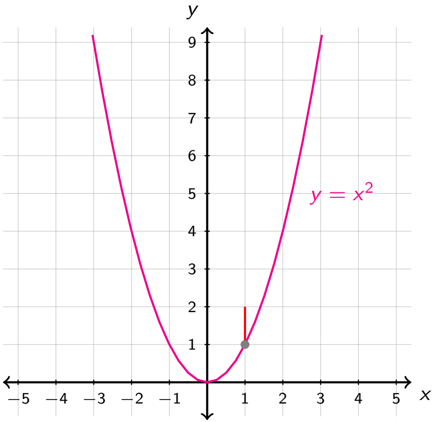

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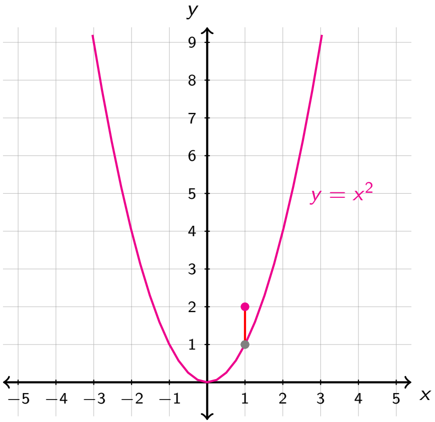

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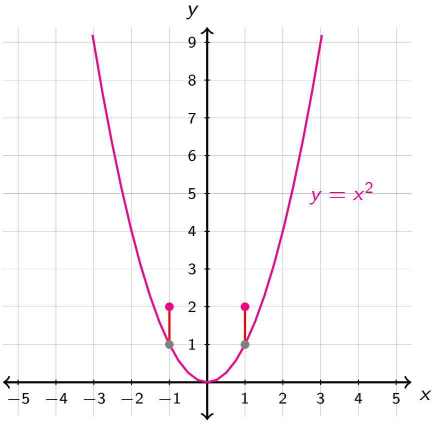

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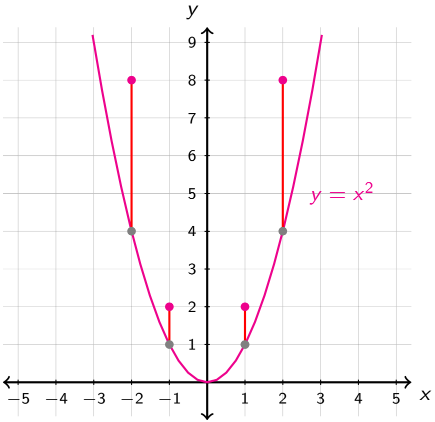

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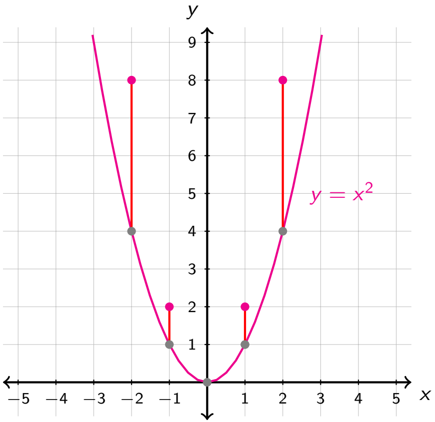

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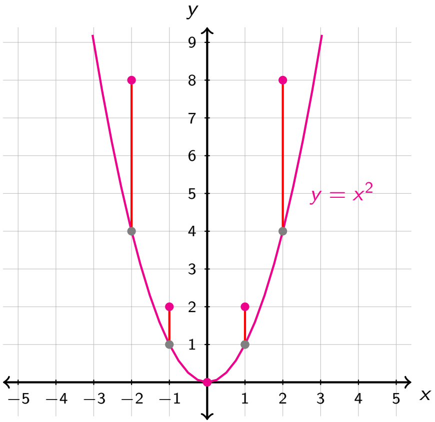

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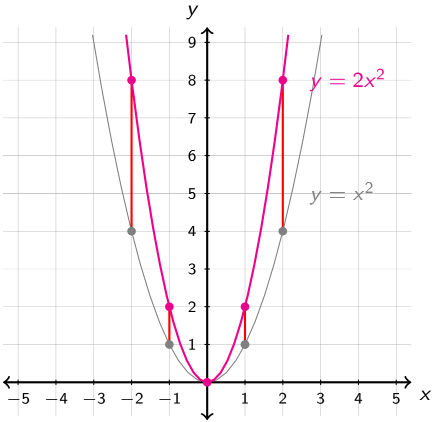

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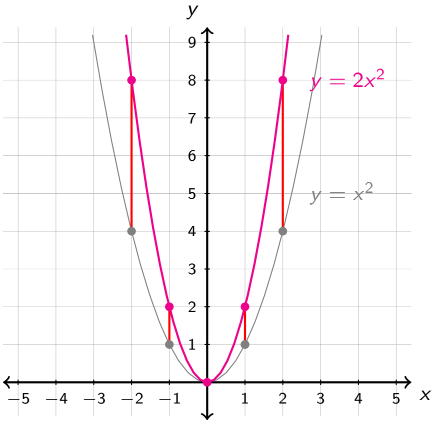

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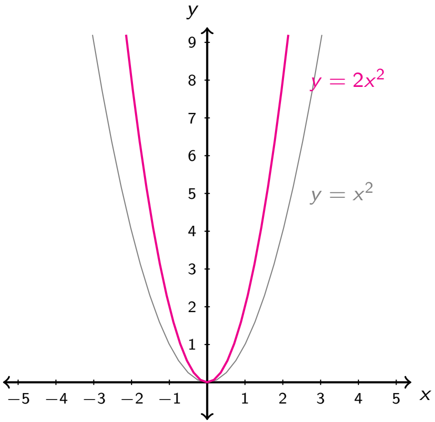

To produce $y = 2x^2$, each point on $y = x^2$ has had its height doubled.

We say that $y = x^2$ has been vertically dilated by a factor of two.

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To dilate a function horizontally by a factor of $\frac{1}{a}$, we multiply x by a.

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To find the graph of $y = (2x)^2$ we horizontally dilate each point on the graph of $y = x^2$ by a factor of $\frac{1}{2}$.

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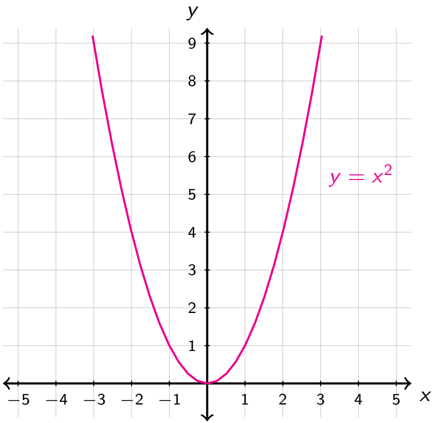

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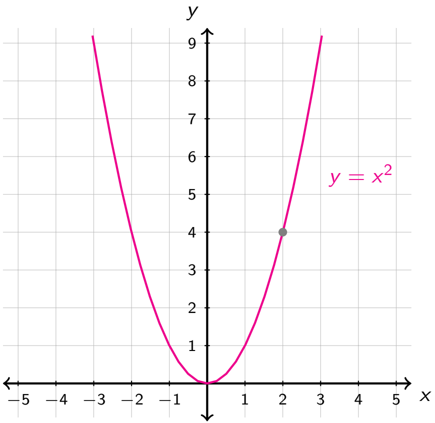

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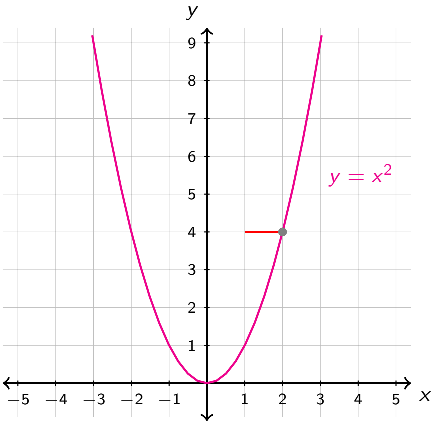

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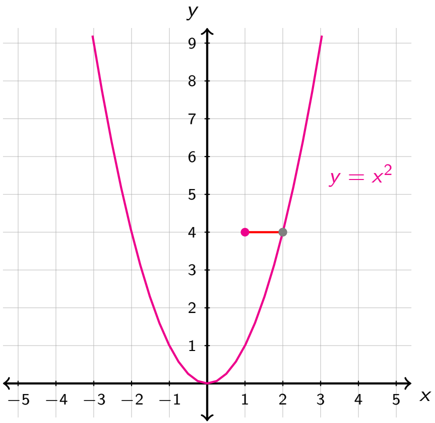

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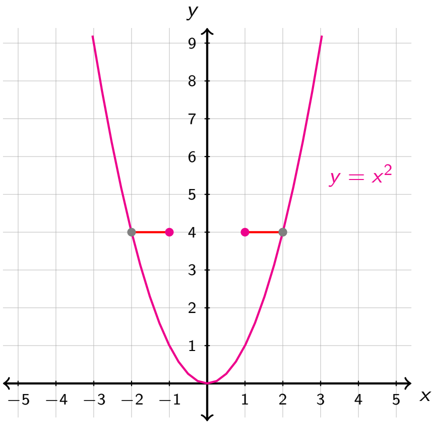

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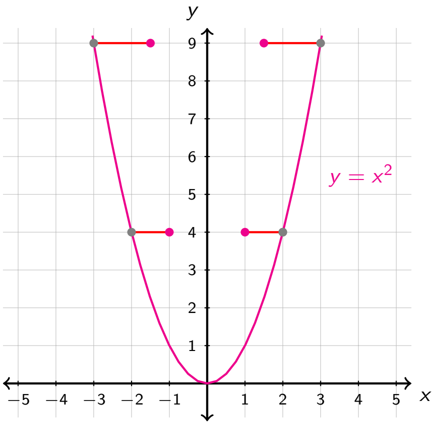

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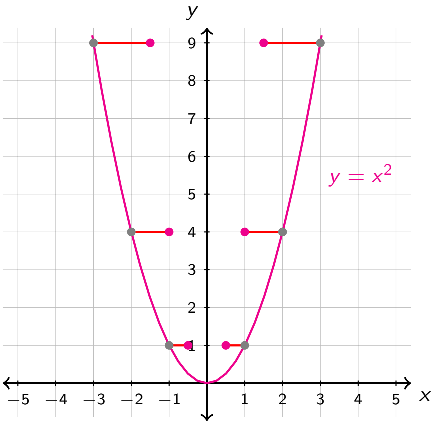

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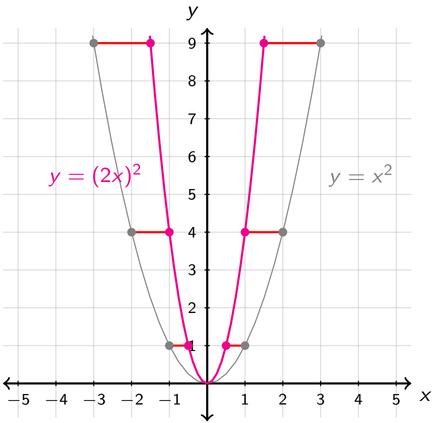

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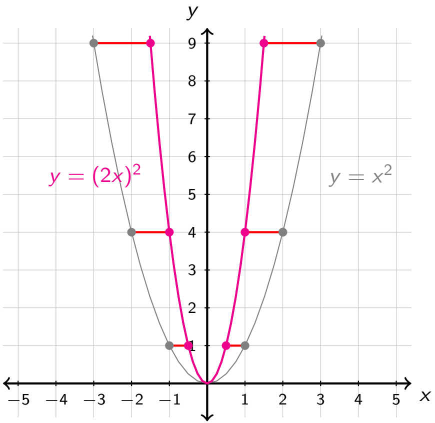

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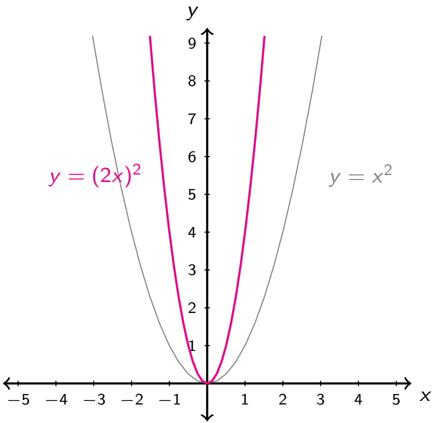

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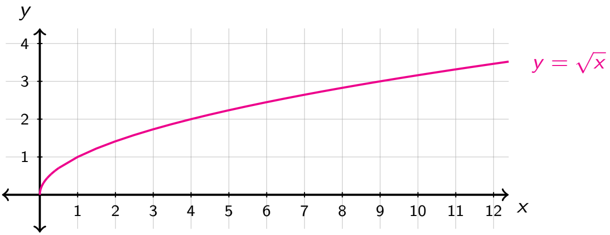

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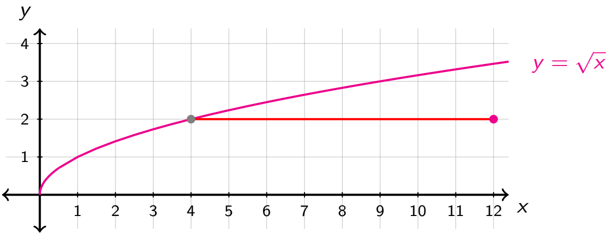

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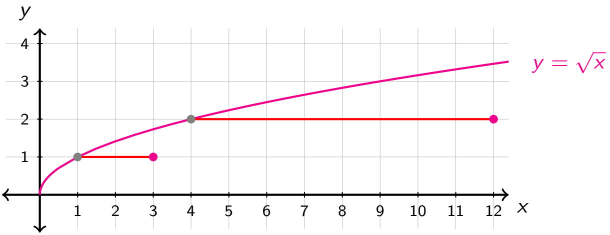

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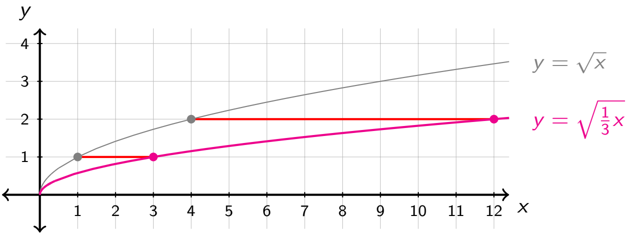

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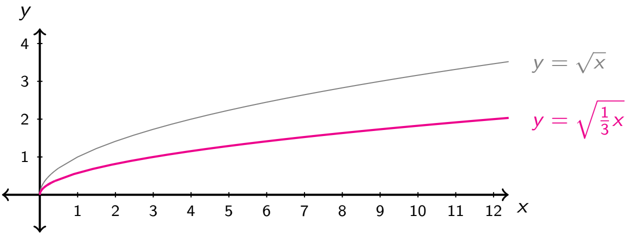

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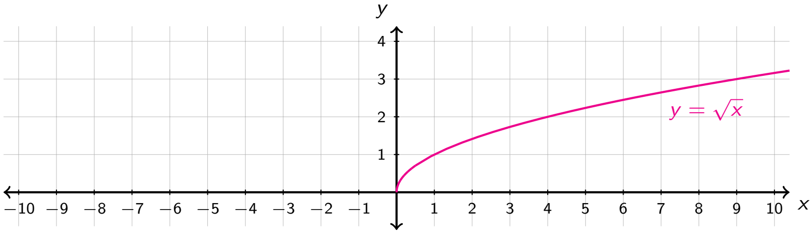

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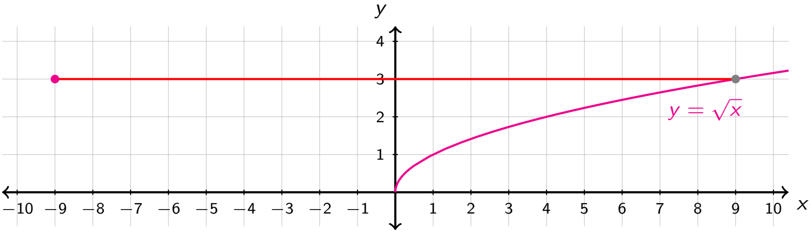

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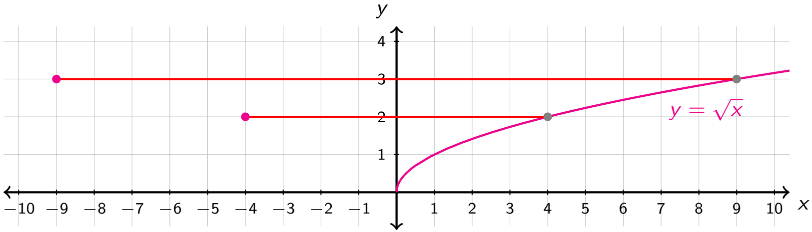

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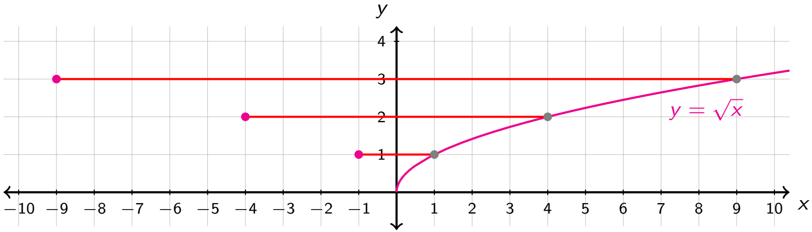

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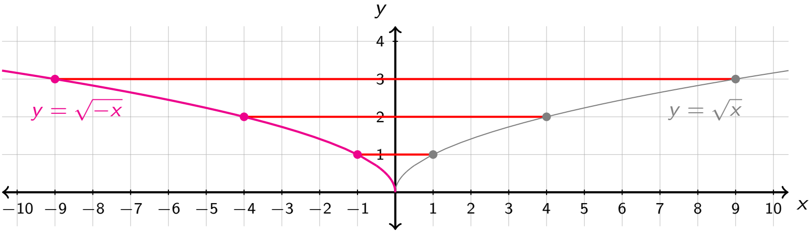
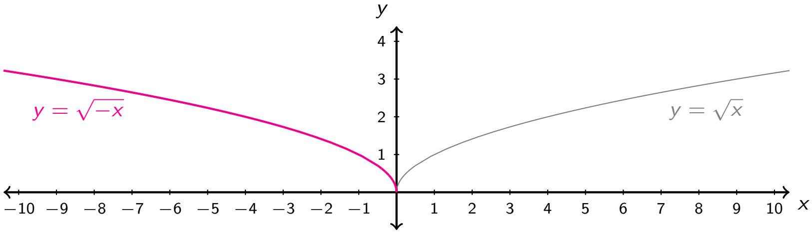


Reflections

To find the graph of $y = \sqrt{-x}$ we reflect each point on the graph of $y = \sqrt{x}$ horizontally (ie. use the y-axis as a mirror).



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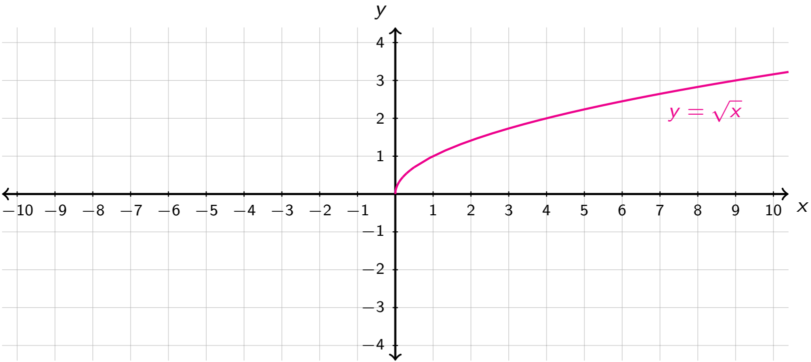

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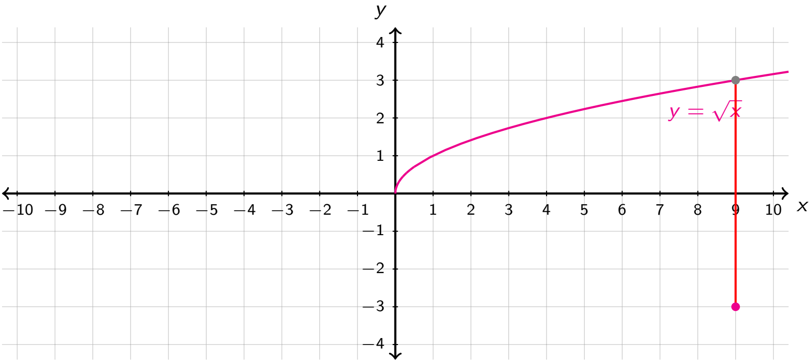

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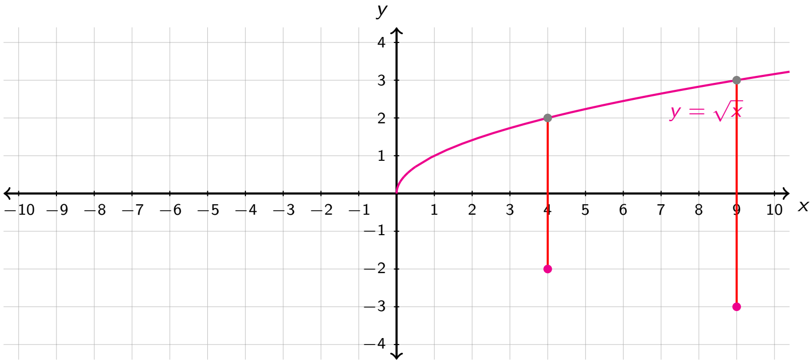

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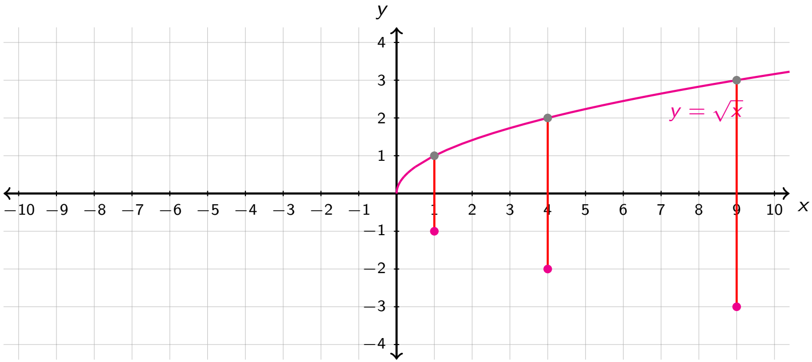

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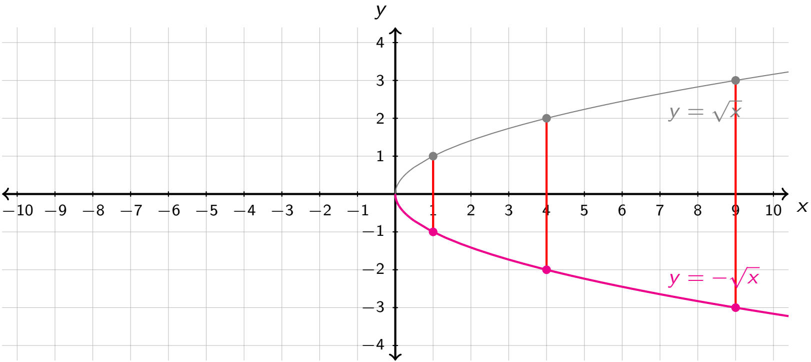

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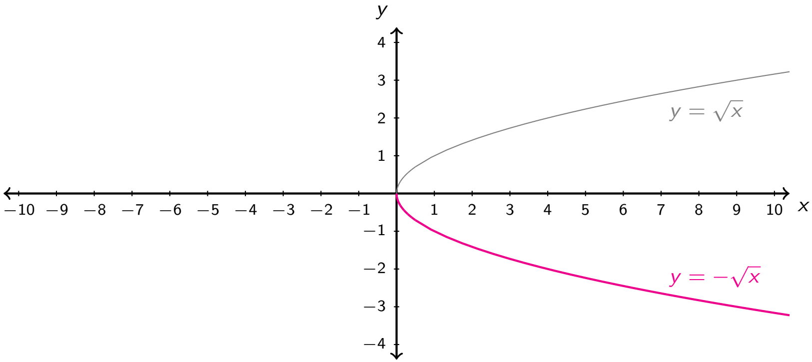

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Reciprocals: $y = \pm \dfrac{a}{\pm bx \pm c} \pm d$

General Transformations

You **need to** read a transformation in this order:

1. Horizontal Translation

2. Horizontal Dilation

2. Reflection about y-axis

3. Reflection about x-axis

3. Vertical Dilation

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(In short, *horizontal* transformations first with *translation* first, then reverse the order for vertical transformations.)

General Transformations: Example

Let's apply these transformations to $y = x^2$ and see if we get the graph for our original example:

$$y = f(x) = -2(-\tfrac{1}{3}x + 2)^2 + 3.$$

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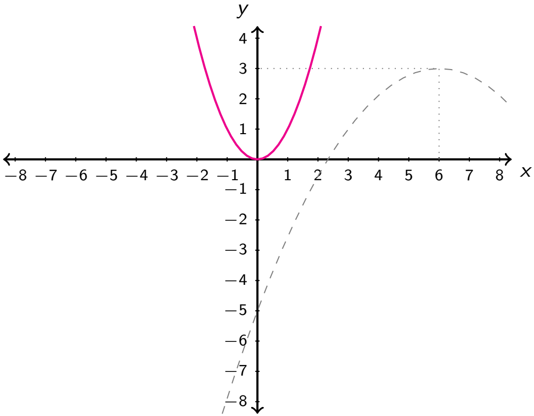

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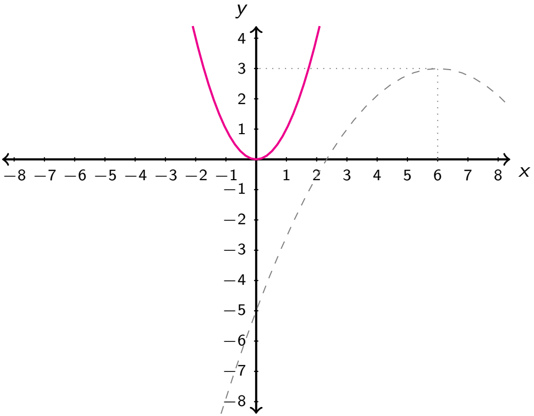

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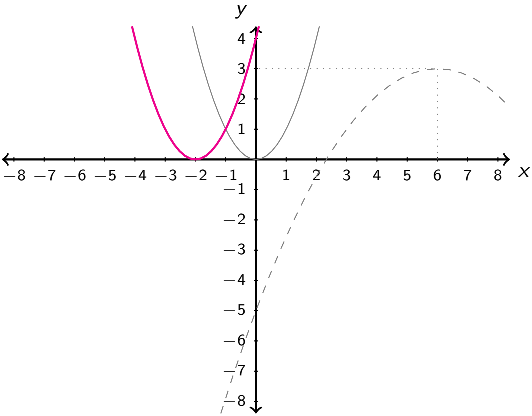

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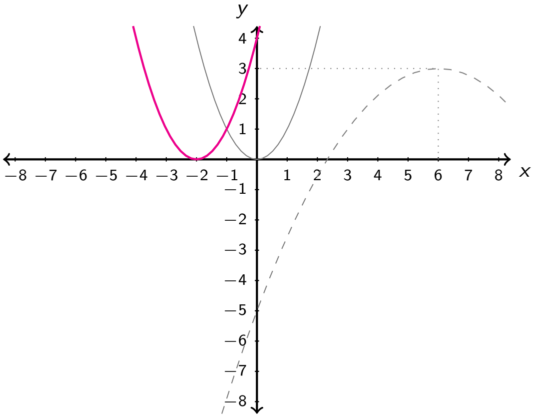

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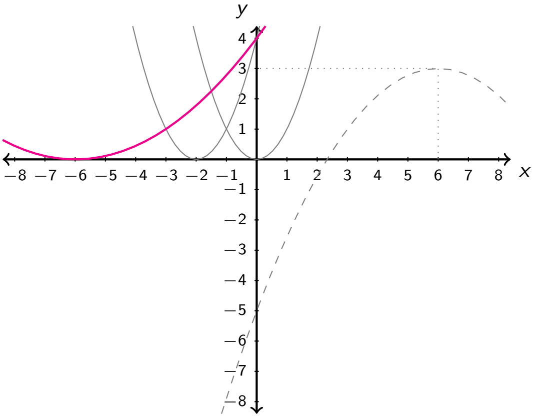

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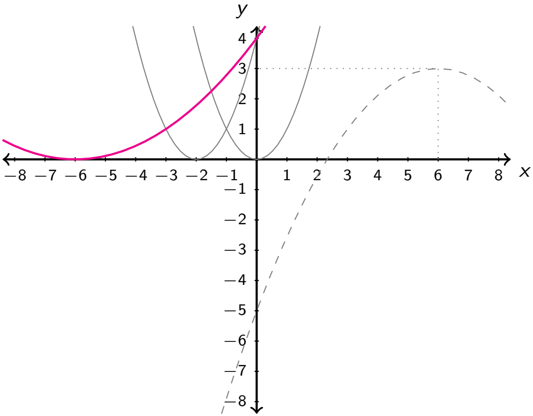

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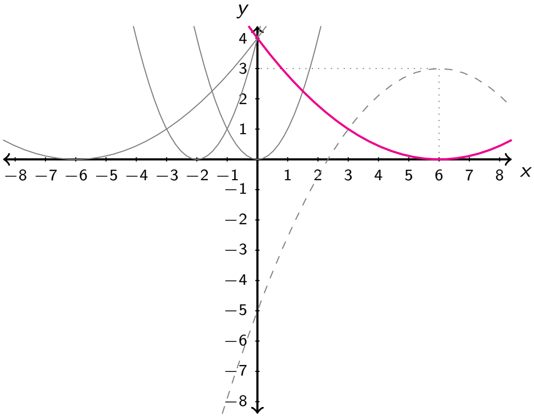

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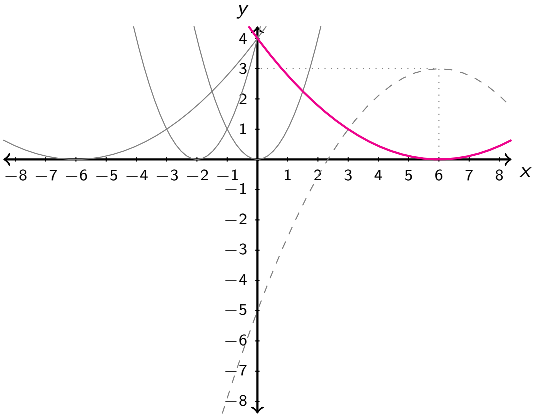

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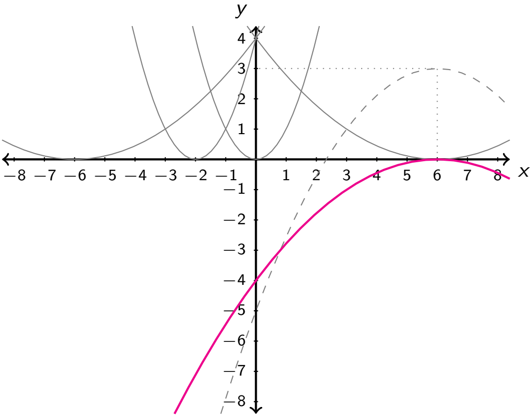

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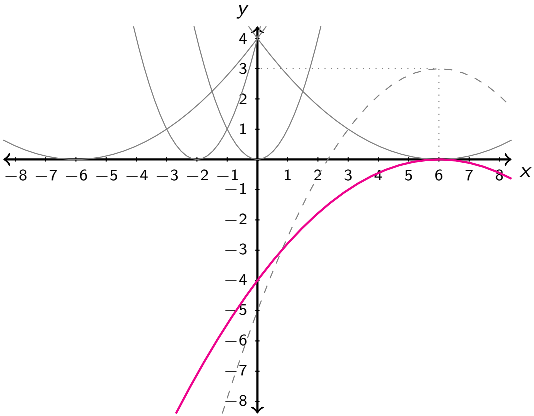

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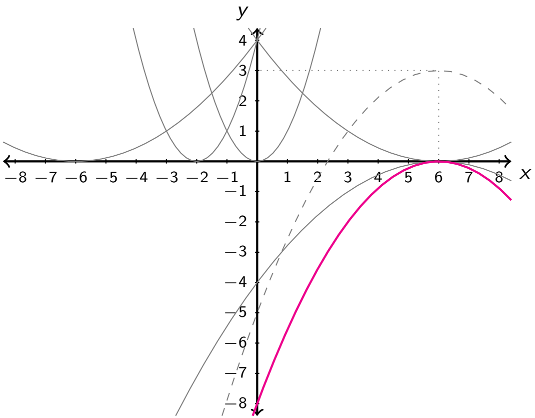

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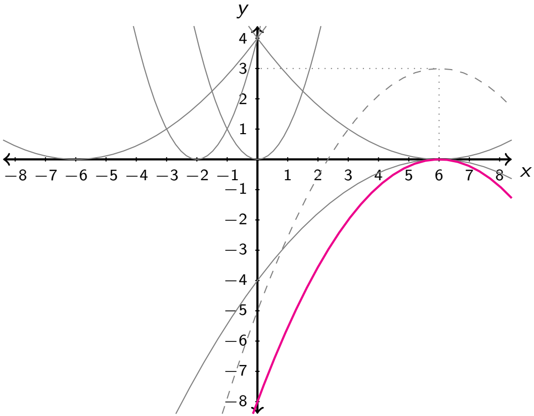

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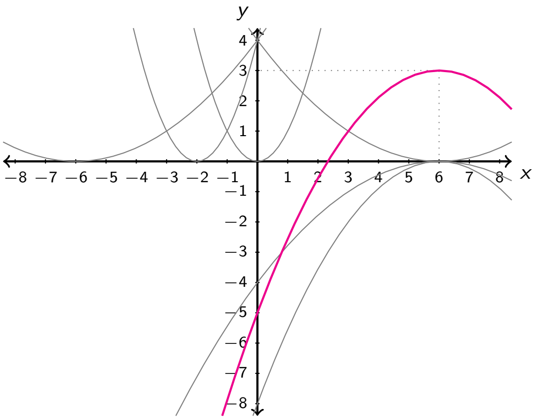

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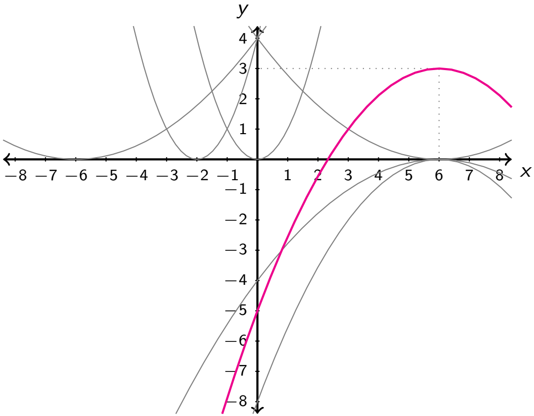

Using STUDYSmarter Resources

This resource was developed for UWA students by the STUDY*Smarter* team for the numeracy program. When using our resources, please retain them in their original form with both the STUDY*Smarter* heading and the UWA crest.

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