23/10/15

Ex 12c Q10

USE PROOF BY INDUCTION TO PROVE THAT $1 \times 2 \times 3 \ldots \times n \geqslant 3^n$ FOR ALL INTEGER VALUES OF $n > 6$

REQUIRED TO PROVE

$$n! \geqslant 3^n \text{ FOR ALL } n > 6$$

1) TEST AND ESTABLISH THE BASE CASE, $n = 7$

$$RHS = 3^7$$
$$= 2187$$
$$LHS = 7!$$
$$= 5040$$
$$LHS \geqslant RHS$$

$\therefore$ THE STATEMENT IS TRUE FOR $n = 7$.

2) WE ASSUME $k! \geqslant 3^k$ IS TRUE FOR ALL $k > 6$

3) REQUIRED TO PROVE

$$(k+1)! \geqslant 2^{k+1}$$

BY ASSUMPTION,

$$k! \geqslant 3^k$$
$$(k+1)(k!) \geqslant 3^k (k+1)$$
$$(k+1)! \geqslant 3^k (k+1) \quad ①$$

(BUT: $k > 6$
$\therefore k+1 > 3$)

$\therefore (k+1)! \geqslant 3^k \times 3$

* SEE BELOW

$$\underline{(k+1)! \geqslant 3^{k+1}}$$

AS REQUIRED.

4) $\therefore$ BY PMI, THE STATEMENT $n! \geqslant 3^n$ IS TRUE FOR ALL $n > 6$.

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THE "SUBSTITUTION" OF 3 FOR $(k+1)$ IS ALLOWED BECAUSE $(k+1) > 3$.

THUS $3^k(k+1) > 3^k(3)$

AND THE RHS OF THE INEQUATION AT ① (PREVIOUS COLUMN) BECOMES "SMALLER", ENSURING

$$(k+1)! \geqslant 3^k (k+1)$$

BECOMES "MORE TRUE", AS IT WERE, AS $3^k(3)$ IS SMALLER THAN $3^k(k+1)$

THE ONLY REASON WE SUBSTITUTE 3 IS SO WE CAN OBTAIN THE EXPRESSION $3^k(3)$

$$= 3^{k+1}$$

AS REQUIRED BY THE RHS OF OUR $(k+1)$ CASE.